Physics 4BL: Electricity and Magnetism
Lab manual

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Introduction

The objective of this course is to teach electricity and magnetism (E&M) by observations from experiments. This approach complements the classroom experience of Physics 1B,C where you learn the material from lectures and books designed to teach problem solving skills. Historically, E&M evolved from many observations that called for a theoretical explanation. It is a great achievement that all classical E&M phenomena can be explained by four equations, the so-called Maxwell's equations. This laboratory course is designed to perform experiments showing the validity of these equations.

In the laboratory, you will have different experiences than in the classroom. In the real world, there are no point sources, no infinities, all measurements have errors, and sometimes things don't work out as expected. A broken instrument or wire can be as frustrating and time consuming as trying to solve a seemingly impossible homework problem. You will have to learn patience and persistence to make good measurements.

For solving theoretical problems you first need to learn the appropriate mathematical tools. For performing experiments you first need to become familiar with measurement tools, called instruments. These include multimeters, oscilloscopes, signal generators, a Gaussmeter, digital scale, power supplies and computers. The first lab session is devoted to acquaint you with modern digital data acquisition methods. It is expected that you are already familiar with Personal Computers (PCs) and basic software such as spreadsheets. In the lab you will record your data to file, copy the files to USB sticks and evaluate the results at home or a computer lab.

For each experiment you are supposed to write a laboratory report. It should contain a very brief description of your experiment (no need to copy the lab manual), the data obtained (usually in the form of graphs) and evaluations such as line, surface and volume integrals, curve fitting, circuit analysis, and any questions raised in the lab manual. The report should be written concisely in a scientific language; it is not an essay where you admire the beauty of science or express your frustrations with the equipment. The TA has no time to read excessively long lab reports. He/she will only look for correct answers and understanding of the results. The reports are due a week after the experiments have been done. Data are shared within an experimental team but the reports should be written individually. Copying other reports constitutes plagiarism and will be reported to the Dean of Students with unpleasant consequences. Bring a personal notebook to the lab to keep a record of what you did. The TA will grade it as well as your lab performance.

I Laboratory 1: Uncertainty and statistics

I.1 Background Information

In this laboratory exercise you will learn about experimental uncertainty, and how we quantify this uncertainty in the values that are measured. In subsequent laboratory exercises you will need to use the methods of error analysis demonstrated here to analyze your uncertainty. You will be provided 1 laboratory session to complete the measurements, and a report summarizing your results will be due the following week.

I.1.1 Uncertainty in Measurements

Any experimentally measured quantity x cannot be known exactly due to error in our measurement. Therefore, when a measured quantity is reported, for example in a scientific journal or an academic lab report, one presents the measured quantity as

$$x = x_{best} \pm \delta x,$$

where x_{best} is our best guess at the quantity x, and δx quantifies the uncertainty in our measurement. Uncertainties in measurement are often classified into two distinct categories, systematic and random. The measurements performed in this laboratory introduce us to analysis methods for treating the latter type, which is statistical in nature. As a result, our confidence level on the outcome is improved by making more measurements, and the exercises here demonstrate how to treat this aspect quantitatively. It is worth emphasizing that appreciating the impact of random uncertainties is fundamental to the process of measuring any physical quantity. Whether it seems relevant for a given situation is quantitative: is the confidence level in the measured outcome small or large when compared to what is needed?

Lets assume N measurements of x are made, yielding values $x_1, x_2, ..., x_N$. A large number of such measurements, called an ensemble, has certain useful properties such as the mean value, denoted by $\overline{x}$, and the spread of the values around it, called the standard deviation, denoted by σ_x. In nearly all situations our best estimate of x is the mean value of these measurements:

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{1}$$

As more measurements are made, we expect that our measured mean value $\overline{x}$ will approach the "true" value X, which is defined as the mean value if an infinite number of measurements were made. That is,

$$X = \lim_{N\to\infty} \overline{x} = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} x_i.$$

What we seek is a quantitative measure of how far the measured mean $\overline{x}$ may deviate from the "true" value X. Standard practice is to use a quantity called the standard deviation of the mean, denoted by $\sigma_{\overline{x}}$ as such a measure. Then, when one publishes (as in your lab reports for this class) a measured quantity, it is written as

$$x = \overline{x} \pm \sigma_{\overline{x}}$$

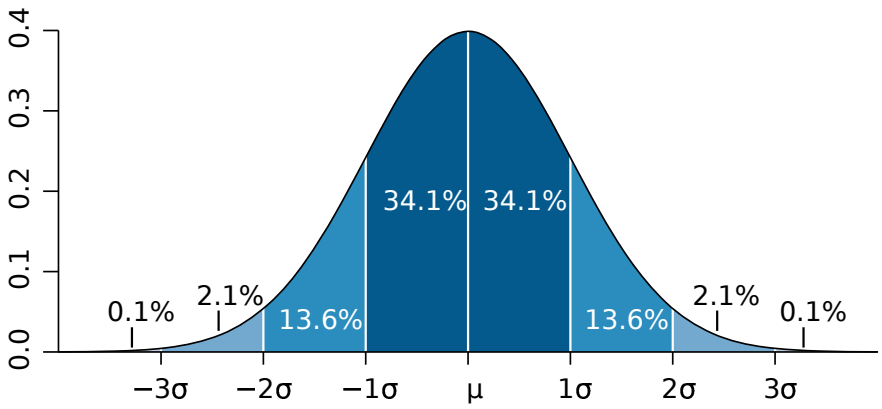

Figure 1:

signifying that we are confident X, the quantity we ultimately want to know, lies within some range of values around our measured $\overline{x}$, due to random error. Standard deviation is based on approximating the data distribution as a Normal or Gaussian distribution. This distribution tells us how many points fall into a standard deviation width, as shown by Figure 1. A smaller value of the standard deviation means that our data is closer to our average, thus improving the accuracy of the average value we measured. In the first part of the present laboratory, you will examine how to calculate our measure of random error, called the standard deviation of the mean, and how it is affected by changing the ensemble size, N.

When performing an experiment, we seek to determine quantities that are measured directly, and also quantities that are *calculated* from something that is directly measured. The fact that error exists in the latter type is referred to as **propagation of error**, and you will study this in the second part of the present laboratory. Consider a quantity F that is a function of 2 directly measured quantities, x and y:

$$F = f(x, y)$$

Since our measured quantities x and y are known only to within some range due to random error: $x = \overline{x} \pm \sigma_{\overline{x}}$, $y = \overline{y} \pm \sigma_{\overline{y}}$, then our calculated quantity must also be known only to within some range: $F = \overline{F} \pm \sigma_{\overline{F}}$. We wish to know how to determine this uncertainty in F due to the random error in measuring x and y. This is calculated by:

$$\sigma_{\overline{F}} = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\sigma_{\overline{x}})^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\sigma_{\overline{y}})^2} \tag{2}$$

which is commonly called the error propagation equation (the derivation of this equation can be found in chapter 5 of Taylor). In the second part of the present laboratory, you will examine two examples with two different functions: addition and division.

I.1.2 Experimental Setup

In the present laboratory, we sample a fluctuating voltage at repeated time intervals with a computerized digital voltmeter. In this way, we easily generate a large ensemble (e.g., $N = 10^5$) of random data. Fluctuating voltages arise from various sources. For instance, the thermal motion of electrons in conductors produce variations in the potential difference between two points, and

the discreteness of the electronic charge leads to current fluctuations. Here we use a semiconductor device called a Zener diode, which is driven in such a way that a relatively large, fluctuating current is produced. In turn, the current is converted to a potential difference by series connecting the diode to a resistor. The block diagram in Fig. 1 shows the basic components producing the large voltage noise amplitude.

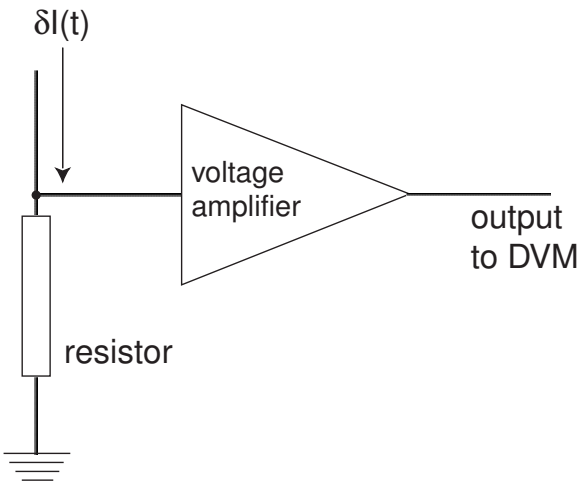

Figure 2: A fluctuating current $I(t)$ is converted to a voltage by the resistor, and then amplified.

In this lab course, you will use a myDAQ (Data Acquisition device) as a digital voltmeter (DVM). The digitizer inputs are red + and black -, and are named CH0 and CH1 in figure 3. The voltage difference is measured between the positive and negative terminals. The negative terminal is often connected to ground, which is essentially zero voltage. The LabView software used to control the ADC, called "4BL Menu" located on the computer's desktop, allows you to analyze data; feel free to explore the options during part 1 of the lab. The software also allows you to save measured data to file, which you can open in Microsoft Excel. To complete your analysis for a given lab, you should copy the Excel files to a USB flash drive. You can find the myDAQ User Manual at http://www.ni.com/pdf/manuals/373060e.pdf

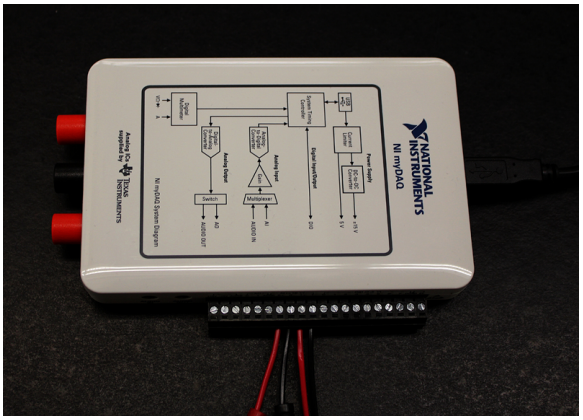

Figure 3: Analog to Digital Converter (ADC) to measure voltages with the computer.

The circuit box used in this experiment is illustrated in Fig. 4. To power the circuit, apply $30V$ to the two terminals (one red and one black) on the left side of the board (see figure 5).

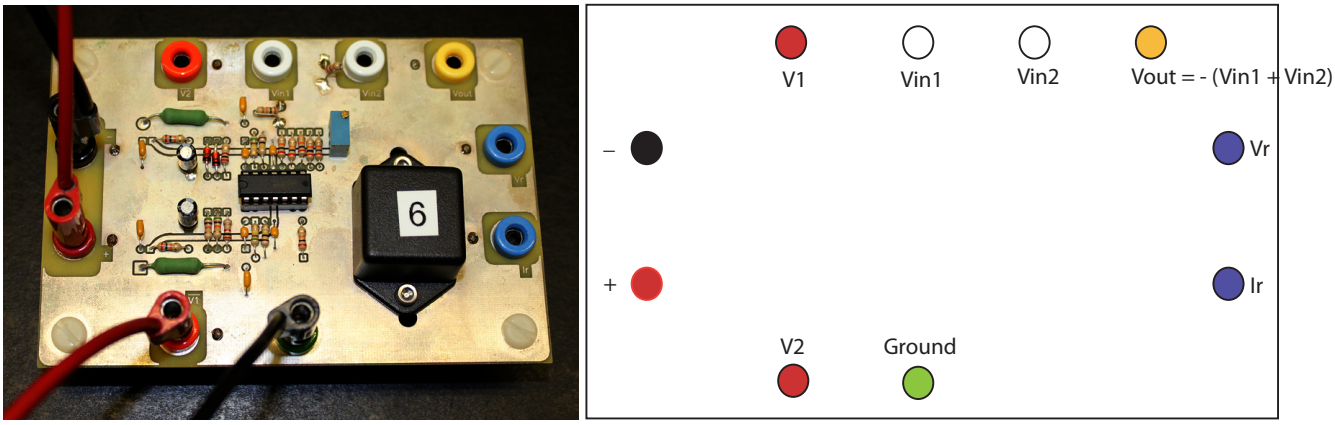

Figure 4: Circuit board and connections needed for the measurements.

In the middle portion of the circuit, there are two red terminals, which provide two independent, random voltages ($\tilde{V}_1$ and $\tilde{V}_2$). The distribution of the random voltages can be very well approximated as a Gaussian form with zero mean and a certain standard deviation. On the upper right corner of the board is an adding circuit with an output of $V_{out} = -(V_{in1} + V_{in2})$, used in the second part of this lab. In the lower right-hand corner is a small circuit used to study the error propagation in Ohm's Law (part 3). The voltage input is V_R, and the output is I_R. The resistor in between is special in that its resistance depends on the external light intensity, and is known as a photo-resistor. A light-emitting diode (LED) controls the resistance, and the voltage input for the LED is V_{in2}. The black canister prevents the room lights from affecting your measurements.

I.2 Procedure and Measurements

I.2.1 Vary the ensemble size

The purpose of this section is to learn how to calculate the measure of uncertainty, called the standard deviation of the mean, and to quantify the impact of the ensemble size on this measurement uncertainty. Intuitively, we take it for granted that how well the mean value is established will improve with more measurements. The random uncertainty, quantified by the standard deviation of the mean, decreases with size N of the ensemble as $1/\sqrt{N}$, which is what we would like to demonstrate.

- Connect 30V from the power supply to the 30V input on your circuit board, as shown in the figures.
- Apply a single noise voltage $\tilde{V}_1$ on the circuit board to the first input channel (CH0) of the ADC. Don't forget to connect the ground. Make the connections with the long red and black banana cords as shown in the figures. The setup is shown in figure 5.
- In the "4BL Menu" software program found on the computer's desktop, Choose the program Acquire Waveform (1 Channel) from the main menu. Use six different sample sizes N = 10, 100, 500, 1,000, 5,000 and 10,000 with a sampling rate at 1,000 points/second. For each value

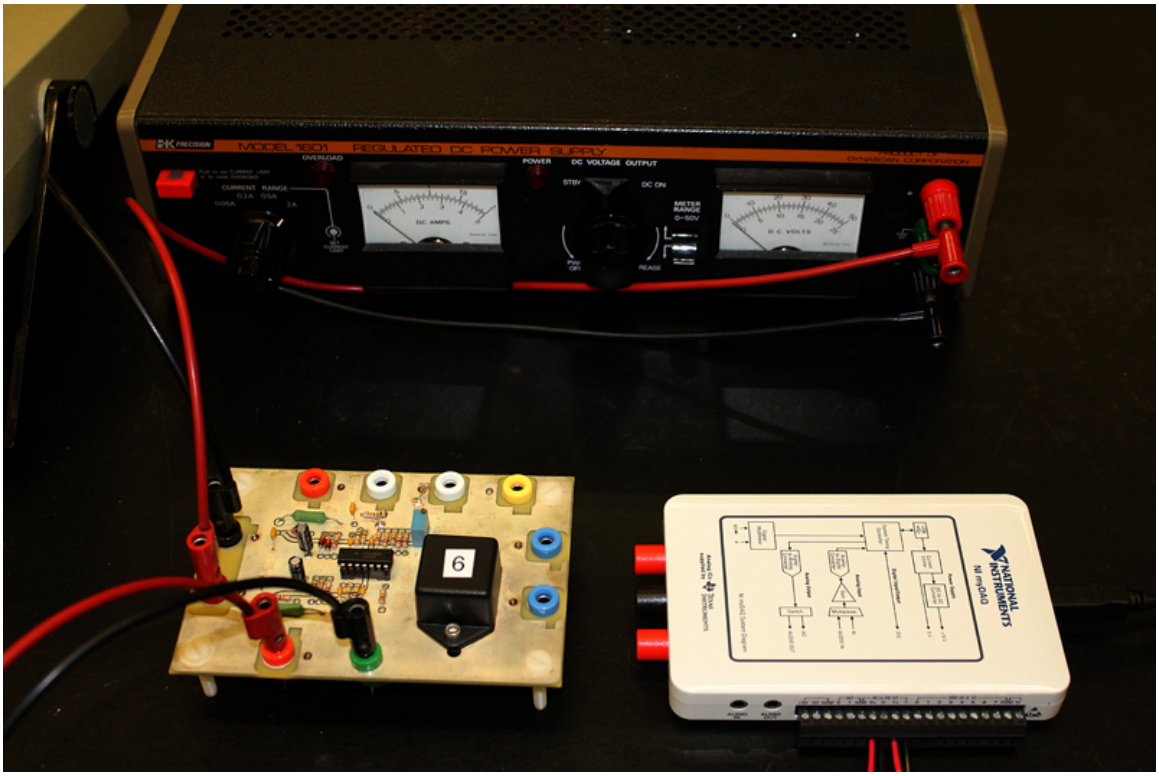

Figure 5: Setup for part 1.

of N, collect 10 ensembles, giving you a total of 60 data sets. Thus, for N=500 for example, you will have ten mean values, which we can denote as $\overline{v}_{500_1}$, $\overline{v}_{500_2}$,..., $\overline{v}_{500_{10}}$ and also 10 standard deviations, which we can denote as σ_{500_1}, σ_{500_2},...,$\sigma_{500_{10}}$.

- After each measurement, click the Statistics button and record the mean and standard deviation in an Excel file.
- while performing these measurements, take note of how much the mean values for a given N vary. The 10 mean values for N=10 should vary substantially, while the 10 mean values for N=10,000 should be quite similar. The standard deviation of the mean, which is our measure of random experimental uncertainty, is calculated just as its name suggests; calculate the standard deviation of the 10 mean values for each N. Thus, you will have six such quantities, $\sigma_{\overline{V}_{10}}$, $\sigma_{\overline{V}_{100}}$, $\sigma_{\overline{V}_{500}}$, $\sigma_{\overline{V}_{1000}}$, $\sigma_{\overline{V}_{5000}}$, $\sigma_{\overline{V}_{10000}}$.

This process of taking multiple ensembles and finding the standard deviation of their mean values is a bit tedious. Statistical theory allows us to calculate this quantity by only taking *one* ensemble, by using the relation:

$$\sigma_{\overline{V}_N} = \frac{1}{\sqrt{N}}\sigma_{N_i} \tag{3}$$

Here, σ_{N_i} is the standard deviation for a single ensemble (you have 10 of them for each N). In the analysis section, you will verify this relation. However, you can use this knowledge for the remaining sections of the experiment and *only take one ensemble from now on.*

I.2.2 Addition of two random voltages

In this part, we will test error propagation using a simple function of addition. We will have two directly measured quantities, $\tilde{V}_1$ and $\tilde{V}_2$, which will have uncertainties $\sigma_{\overline{V}_1}$ and $\sigma_{\overline{V}_2}$, and we want to calculate their sum

$$V_{sum} = -(V_1 + V_2)$$

Since the measured quantities exhibit uncertainty, we have to attribute random uncertainty to our calculated quantity using the error propagation equation. Plugging this simple function into equation 2 gives

$$\sigma_{\overline{V}_{sum}} = \sqrt{\sigma^2_{\overline{V}_1} + \sigma^2_{\overline{V}_2}} \tag{4}$$

While we also expect that

$$-\overline{V}_{sum} = \overline{V}_1 + \overline{V}_2, , \tag{5}$$

In this part, you will use an adding circuit to measure $\tilde{V}_1$, $\tilde{V}_2$ and *also* V_{Sum}. Thus, you will be able to calculate the uncertainty in V_{Sum} as well as measure it, so you can compare your calculated and measured values.

- Connect the random voltage source $\tilde{V}_1$ to the V_{in1} of the adding circuit, and the random voltage source $\tilde{V}_2$ to the V_{in2} of the adding circuit using short cords. Connect the $\tilde{V}_2$ signal to the second channel of the ADC (CH1) in the same way as CH0 is connected to $\tilde{V}_1$ (from part 1).
- In the 4BL Menu, select Acquire Waveform (2 channels). Use the same sample rate as above, and acquire 10,000 points. Measure the mean and standard deviation of each noise signal.
- In your notebook, record the mean and standard deviations measured by the two channels.
- Connect V_{out} of the adding circuit to the ADC (CH0). Use the same sample rate as above, and acquire 10,000 points. Measure the mean and standard deviation of the sum for a single ensemble.
- In your notebook, record the mean and standard deviation of the sum.

I.2.3 Quotient of two random variables

Finally, consider the error propagation for the product/ratio of two random variables. A straightforward instance where this may arise is Ohm's law, $V = IR$, an equation we will use many times throughout the quarter. If the quantity of interest is the current (I = V/R), then we need to account for the errors in both the voltage and resistance in attributing error into the calculated current. Putting this equation into the error propagation equation gives (after some algebra)

$$\left(\frac{\sigma_{\overline{I}}}{\overline{I}}\right)^2 = \left(\frac{\sigma_{\overline{V}}}{\overline{V}}\right)^2 + \left(\frac{\sigma_{\overline{R}}}{\overline{R}}\right)^2. \tag{6}$$

(Derive this equation for yourself.)

I.3 Analysis and Report Guidelines

I.3.1 Vary the ensemble size

You have recorded 10 mean values and 10 standard deviations for 6 different ensemble sizes. Let us denote these mean values by $\overline{V}_{N_i}$, and these standard deviation values by σ_{N_i}, where N refers to the sample size, and i runs from 1 to 10 since we collected 10 data sets for each N. Notice that the 10 mean values for N=10 vary quite a bit from each other, whereas the 10 mean values for N=10,000 are similar. We quantify this variation with the standard deviation, which will be our quantification of the random uncertainty.

- In an Excel file, calculate the standard deviation of the 10 mean values for each N. This quantity is the standard deviation of the mean mentioned earlier, $\sigma_{\overline{V}_N}$. You will have six such quantities, one for each value of N. Our premise is that the more measurements we take, we reduce our random uncertainties and thus the standard deviation of the mean should decrease.
- Create a plot with $\sigma_{\overline{V}_N}$ on the y-axis and N on the x-axis. Use a logarithmic scale on both axes, which can be accomplished in the "chart options in excel". This shows that our random uncertainties are reduced the larger our ensemble size, and how to calculate the standard deviation of the mean, which is a measure of these random uncertainties. However, the process of taking multiple ensembles and finding the standard deviation of their mean values is a bit tedious. Statistical theory allows us to calculate this quantity by only taking *one* ensemble, by using the relation:

$$\sigma_{\overline{V}_N} = \frac{1}{\sqrt{N}}\sigma_{N_i} \tag{7}$$

- To show this relation, plot on the same graph as above $\sigma_{N_i}/\sqrt{N}$ on the y-axis and N on the x-axis (remember that σ_{N_i} were the very original standard deviations you recorded. You have ten values for each N, but for this plot just use one of them). Include this plot in your lab report and discuss your results.

I.3.2 Addition of two measured quantities

You have recorded the mean value of two voltage sources V_1, V_2, and also their standard deviations, σ_{V_1}, σ_{V_2}. Calculate their sum $\overline{V}_{sum}$ using the two measured values. Additionally, you *measured* the sum using the adding circuit. Compare your measured and calculated values for $\overline{V}_{sum}$.

When reporting a calculated quantity, in this case $\overline{V}_{sum}$, one needs to report the uncertainty in its value due to the random error in the measured quantities it is calculated from. We do this with the error propagation equation presented in the introduction section.

- Calculate the uncertainty of the measured quantities V_1 and V_2 using the method from section 1: $\sigma_{\overline{V}_1} = \sigma_{V_1}/\sqrt{N}$, and $\sigma_{\overline{V}_2} = \sigma_{V_2}/\sqrt{N}$.
- then calculate the uncertainty $\sigma_{\overline{V}_{sum}}$ in the calculated quantity $\overline{V}_{sum}$, using the error propagation equation.

- However, you have also *measured* $\overline{V}_{sum}$, thus you can determine the measured uncertainty, i.e. calculate the standard deviation of the mean of $\overline{V}_{sum}$: $\sigma_{\overline{V}_{sum}} = \sigma_{V_{sum}}/\sqrt{N}$. Compare this measured uncertainty with the number you calculate using the error propagation equation.

I.3.3 Quotient of two measured quantities

Question Assume that a quantity F is calculated from two measured quantities x and y from the function

$$F = xy.$$

Let the uncertainty in x be $\sigma_x = 0.1$ while the uncertainty in y is $\sigma_y = 0.001$ with $\overline{x} = \overline{y} = 1$. Calculate the uncertainty in F using the error propagation equation. Does excluding the term with σ_y make any substantial difference? This is an example when one measured quantity is the dominant source of error.

I.3.4 Lab report guidelines

Your lab report should contain the following sections

1. **Header**
   - Descriptive title
   - Name, Date, and class information
   - Name of your lab station partners (lab partners share data, however each must perform their own analysis and write their own report)
2. **Introduction**

   The introduction section explains, in your own words, the purpose of your experiment and how you will demonstrate this purpose. Try and be as brief as possible, yet still get your point across.
3. **Experimental Results**

   This section should briefly explain what was measured and how the data was collected (diagrams of the experimental set up may be helpful), present the raw data in graphical form (if possible) including uncertainty values with all numbers. Your graph should include labels on both axes that include units. Label your graphs as figure 1, figure 2, etc, so that you can refer to them in your text. In the text, explain the meaning of the variables on both axes and an include an explanation that makes it clear to the reader how to interpret the information displayed on in the plots.

   your report should contain:
   - graph of $\sigma_{\overline{V}_N}$ and $\frac{1}{\sqrt{N}}\sigma_{N,i}$ vs N, using a logarithmic scale on both the x and y axis, in order to display the wide range in values.
   - measured data from section I.2.2.
4. **Analysis**

   this section should present any calculations performed on the raw data, including uncertainties using propagation of errors. Example: if you are testing Ohm's Law $V = IR$, and you

measured V and R, then here you calculate I. If you have a theoretical value for your calculated quantity (I in this example), state whether it is within your uncertainty range. Also, answer any questions proposed in the lab manual.

you should have:

- Comment on how well your data for $\sigma_{\overline{V}_N}$ vs. N agrees with what you theoretically expect, $\sigma_{\overline{V}_{N_i}}/\sqrt{N}$.
- For the addition of two voltages section, calculate the theoretical value for $\sigma_{\overline{V}_{sum}}$ and compare with your measured value. Answer questions posed in section I.3.2
- For the quotient of two voltages section, Answer questions posed in section I.3.3

5. **Conclusion**

   State how your results demonstrate (or fail to demonstrate) the objectives you presented in the introduction section.

II Laboratory 2: DC and AC circuits

II.1 Background Information

In this laboratory exercise, you will examine some basic properties of DC and AC circuits. The exercise will be completed in two laboratory sessions, with the first week set aside for DC circuits and the second for AC circuits. Additionally, you will learn a statistical analysis technique called linear regression which you will use to analyze your data. After completing the exercises during both lab sessions, you will write one lab report describing your work for both sessions, which will be due 1 week after the lab is completed. Some measurements you make will not be required in the lab report, so make sure you comment thoroughly in your notebook to receive credit for completing these portions.

II.1.1 DC circuits

A direct current (DC) circuit is defined as a circuit in which the current flows in only one direction, as opposed to an alternating current (AC) circuit, where the direction of the current alternates periodically with time. A DC circuit is often created by connecting a load, which is the device we wish to run a current through, to the positive and negative terminals of a battery. Once the circuit is closed by creating these connections, charge carriers flow from the positive terminal to the negative terminal of the battery through our load device due to the potential difference created by the battery. First, we will use a single resistor as our load device. Second, we will use combinations of multiple resistors as our load, and finally we will use a device called a diode as our load.

When analyzing a load device, one generally asks: what is current-voltage characteristic of the load? To answer this question, you have to look at an IV curve for the device. The IV curve will tell you most of the properties of how the device functions, and its usefulness in an electronic device. Basically, an IV curve tells you how much current will flow through the device when a specific voltage is applied. By varying the voltage applied across the device and measuring the current through it, one can generate an IV curve. An IV curve for a simple resistor will yield Ohm's Law,

$$I(V) = \frac{V}{R} \tag{8}$$

which states that the current through the device increases linearly as the voltage is increased. The resistance R of the device is defined by this relation, and is a characteristic of the material used to create the resistor, which you will study further at the end of this lab. Your first task will be to create an IV curve for a resistor to verify Ohm's Law.

Next, you will build a circuit with two resistors in series, and then two resistors in parallel. The equivalent resistance of a combination of N resistors in series is given by

$$R_s = \sum_{i=1}^{N} R_i. \tag{9}$$

The comparable expression for N resistors in parallel is

$$\frac{1}{R_\parallel} = \sum_{i=1}^{N} \frac{1}{R_i}. \tag{10}$$

For two resistors, this result simplifies to

$$R_{\|} = \frac{R_1 R_2}{R_1 + R_2}.$$

The second part of today's exercises is to verify these relationships of equivalent resistances.

Next, the ordinary IV behavior of resistors is contrasted with a diode, which is a non-ohmic circuit element. This is a nonlinear device with properties that result from bringing two different semiconducting materials together to form a junction. It has innumerable applications in electronic circuits. The simplest is a rectifier, which passes a current with only a small applied voltage in one direction, but not the other. You will measure an IV curve for a rectifier diode to verify this property. The IV relation we expect is:

$$I(V) = I_0(e^{|e|V/(nk_BT)} - 1), \tag{11}$$

where I_0 is a constant specific to the actual diode under study, k_B is Boltzmann's constant, T is the temperature measured in Kelvins, and $|e|$ is the magnitude of the electronic charge. The constant n is called the ideality factor, which is dependent on the diode's material and fabrication. Typically this value varies from 1 to 2.

Finally, as a simple and fun exercise, you will investigate the variation with temperature of the resistance of a length of copper wire. For simplicity, our exploration of the effect of temperature is limited to two fixed points, namely ambient temperature and that of liquid nitrogen. The resistance R of a particular material depends on both extrinsic details, such as geometry/size of the device, as well as an intrinsic property of the material called the resistivity ρ (or the conductivity $\sigma = 1/\rho$). The resistance of a wire is related to its resistivity and wire geometry by

$$R = \frac{\rho \ell}{A} \tag{12}$$

where ℓ is the length of the wire, and A is its cross sectional area. The resistivity of a material changes with temperature, and you will verify this by measuring the resistance of copper wire at two different temperatures.

II.1.2 AC circuits

To create an AC circuit, we connect our circuit element to a voltage source that varies sinusoidally, causing a current to be driven which alternates in direction with time. In addition to the resistive components studied in part I, an AC circuit also uses reactive components: capacitors and inductors. The capacitance of a capacitor is defined as the ratio of the charge built up on each plate to the voltage difference between the two plates:

$$C = \frac{Q}{V} \tag{13}$$

The unit of measure of capacitance is called the Farad. Differentiating this equation with respect to time yields an equation that relates the current through this circuit element to the voltage applied:

$$I = C\frac{dV}{dt} \tag{14}$$

This shows that current flows through a capacitor only when we have a voltage that varies in time. Thus, if we connect a circuit containing a capacitor to a DC battery, no current will flow in the steady state because the capacitor acts as an open circuit. However, current will flow initially when the circuit is closed, since at the instant the circuit is connected to the battery, the voltage changes rapidly from zero volts to the voltage of the battery.

Consider a simple circuit with a resistor and capacitor in series, called an RC circuit, connected to a battery. At the instant the circuit is connected to the battery, dV/dt is large, and so the capacitor doesn't impede the current flow. The current flow is impeded only by the resistor, and thus the full voltage of the battery is across the resistor. After a certain amount of time, the voltage settles at a constant (the voltage of the battery), all the charges are piled up on the plates of the capacitor, and thus no current flows. Therefore, the current is impeded only by the capacitor, and thus the full voltage of the battery is across the capacitor. This is called charging the capacitor. The initial response where the current is flowing is called the transient period, and the period when the current stops flowing is called the steady state. There is a a characteristic time for the transition between these two periods called the RC time. By using Kirchhoff's Law, Ohm's Law, and equation (13) we can derive the voltage as a function of time across the capacitor. The result is:

$$V_C(t) = V_b \left(1 - e^{-t/RC}\right) \tag{15}$$

where V_b is the voltage of the battery and we have assumed that at time $t = 0$ the circuit is connected to the battery. This equation defines the time scale of transition between the transient and the steady state:

$$\tau = RC.$$

The other reactive component we will be studying is called an inductor, which is characterized by its inductance defined by the relation:

$$V = L\frac{dI}{dt}. \tag{16}$$

The inductance L is measured in the unit Henry. This equation also shows that the voltage across an inductor is large when we have a rapidly changing current. Thus, if we form a circuit with a resistor and inductor in series (called an LC circuit) and at time $t = 0$ connect this circuit to a battery, we will again have a transient period and a steady state period. During the transient period, the current is rapidly increasing from no current (when the circuit is disconnected from the battery) to its maximum current. During this period, dI/dt is large, and thus the entire voltage is dropped across the inductor. In the steady state, the current settles at its steady state value, the current is impeded by only the resistor and thus the entire voltage drop is across the resistor. We can use Kirchhoff's Law, Ohm's Law, and equation (16) to derive the voltage on the inductor as a function of time:

$$V_L(t) = V_b e^{-tR/L} \tag{17}$$

where V_b is the voltage of the battery and we have assumed that at time $t = 0$ the circuit is connected to the battery. This equation defines the time scale of the transition between the transient and steady state periods:

$$\tau = L/R.$$

In the first part of week 2, you will build an RC and an RL circuit, measure the voltage during the transient time period, and calculate the characteristic time constant, verifying equations (15) and (17).

In the second part of week 2, you will build what's called an RLC resonant circuit, and examine the response of this circuit to sinusoidal voltages of different frequencies. You will see that the performance of the circuit depends greatly on the frequency of the driving voltage. However, first we need to review the concept of impedance.

Ohm's Law $V = IR$ states that the current through a resistive component is proportional to the voltage applied. We can generalize this equation to also include reactive components (capacitors and inductors). The result is

$$V = IZ \tag{18}$$

where Z is called the impedance, and the voltage V is assumed to vary sinusoidally: $V(t) = V_0 \sin(\omega t)$. The impedance is a generalized resistance, which takes into account the fact that capacitors and inductors impede the flow of current when we have voltages and currents that vary in time. The impedance is a complex quantity, meaning that it is described using imaginary numbers. The real part of the impedance is called the resistive component, and the imaginary part is called the reactive component. An understanding of our circuits in terms of complex numbers won't be necessary for this course and we will skip a detailed analysis and just state the results.

First, the impedance of a resistor is just its resistance: $Z_R = R$, independent of the frequency ω of the driving voltage. Next, the impedance of a capacitor is given by

$$Z_C = \frac{1}{i\omega C}.$$

The feature to note is that the impedance is proportional to the $1/\omega$. This means that when $\omega \to 0$ the impedance of the capacitor becomes large. This makes sense when considering equation (14) because a small ω means the voltage is changing slowly, dV/dt is small, and a small current is allowed to flow. Conversely, if $\omega \to \infty$, then dV/dt is large, and a large current is allowed to flow. The impedance of an inductor is given by

$$Z_L = i\omega L,$$

which states that the impedance is proportional to ω. This makes sense in terms of equation (16) because a small ω corresponds to a small dI/dt and a large ω corresponds to a large dI/dt.

The circuit you will build to test these ideas is called an RLC resonant circuit, where an inductor, capacitor and resistor are connected in series. The series combination will be connected to a sinusoidal voltage source, and the voltage across only the resistor is the output voltage. The circuit is shown in figure 6. You will vary the frequency of the input voltage and measure the response of the output voltage. At low frequency, we know that the impedance of the capacitor Z_C will be high, and thus since $V_C = IZ_C$ all the voltage will be across the capacitor, leaving zero voltage across the resistor (which is the voltage we are measuring). At very large frequency, the impedance of the inductor will be large, the entire voltage will be across the inductor and again the voltage across the resistor will be small. Somewhere in the middle, the inductor and capacitor will effectively cancel each other out, and the entire voltage will be across the resistor. The frequency at which this occurs is called the resonant frequency, and is determined by

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}.$$

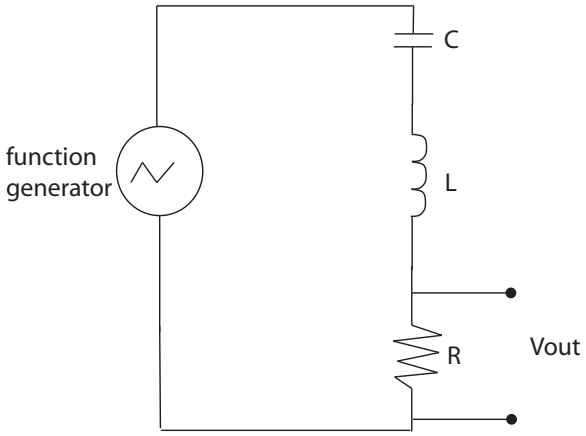

Figure 6: An RLC resonant circuit used in the AC circuits portion of the laboratory. A sinusoidal voltage is applied across the series connection fo the 3 components, and the output voltage is the voltage measured across the resistor.

In the second part of the AC circuits experiment, you will build an RLC circuit and find the resonant frequency by looking at the transfer (i.e. response) function $G(\omega)$ from the input V_{in} to the output V_R:

$$G(\omega) = \frac{V_R}{V_{in}} = \frac{IR}{IZ} = \frac{R}{R + i(\omega L - \frac{1}{\omega C})} \tag{19}$$

As we can see, the response function is frequency dependent and we expect a maximum value when the imaginary part of the denominator vanishes, that means, when $\omega^2 = 1/LC$ which is exactly the resonant frequency. Thus the resonant frequency is easily found by finding the $\max[G(\omega)]$.

II.1.3 Experimental setup

In the present laboratory, you will use a function generator to create various waveforms. The function generator is shown in figure 7. The voltage from the OUTPUT port of the function generator should be sent to the the aluminum break out box, which is provided to convert signals from BNC type connectors to banana plug type connectors. The aluminum connector box is also shown in the figure. The red banana plug ports are connected to the positive terminal of the BNC ports on their respective sides, and the black ports are connected to ground. The yellow ports are not connected to anything which you will need to use to build your circuits.

You are also provided various resistors, capacitors and inductors that you will use to build various circuits. These components have been permanently attached to yellow banana plugs, which you can plug into the top of the connector box to build your circuits. The circuit elements are organized in a clear plastic case which has labeled compartments, so please take care to return the circuit elements into their proper location. To take measurements, you will use the computer and ADC used in the previous laboratory assignment, or a digital multimeter (DVM). At your lab station, you have one of the DVM's shown in the figure. In the second week, you will also use a device

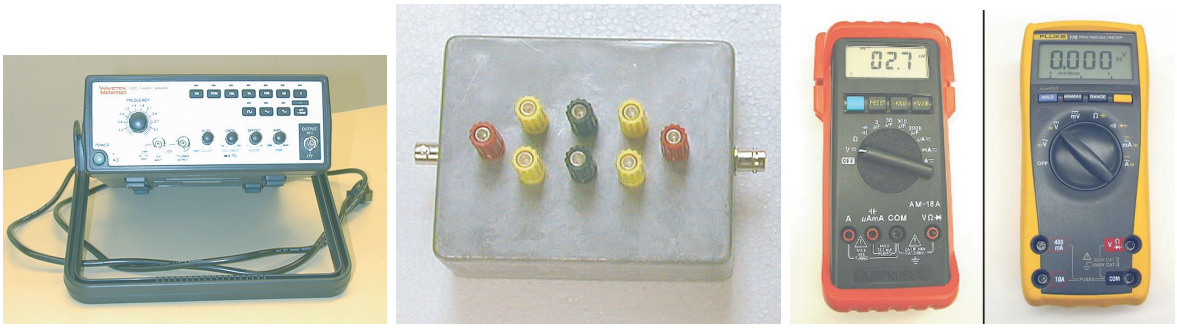

Figure 7: The function generator you will use to create various types of waveforms. The breakout box used to build your circuit. A digital multimeter (DVM) for measuring voltages or resistance.

called an oscilloscope (called a scope for short), which is a device to measure voltages as a function of time. During your laboratory session, your TA will explain how to use the scope.

II.2 Procedure and Measurements

II.2.1 DC Circuits - week 1

Verification of Ohm's Law In this section, you will generate a current vs. voltage (IV) relationship for a resistor R, using the function generator as a source. To do so, we need to vary the applied voltage across the resistor R we are studying, and also measure the current through it. Instead of connecting the resistor to different batteries with varying voltages, we will connect the resistor to the function generator and apply a triangular wave, thus ramping through a range of voltages quickly and easily. Then, we need to measure the voltage V_R across the resistor R we are studying, which we can do with the computer's ADC. We also need to measure the current through the resistor, however our ADC only measures voltages. Thus, we will make a series connection with a "known" resistor r and measure the voltage drop across it with the ADC to determine the current (by dividing the voltage by the resistance r). The circuit you are building is shown in figure 8. **Comment:** It may seem that the argument here is circular: we assume that Ohm's Law applies to the resistor r in order to demonstrate it for R. It is. What we measure are the potential differences on each, and evaluate the ratio. However, the fact that the ratio is constant, independent of V_R and the value of R, makes the claim that Ohm's Law is obeyed stronger that it would first appear.

- Set the function generator output $V_{out}(t)$ to a triangular wave with 10 Hz frequency and amplitude of a few volts. Connect this to the aluminum breakout box using a BNC cord.
- on top of the aluminum breakout box, build a series circuit with a resistor $R \approx 1000\Omega$ and $r \approx 100\Omega$. Measure the resistance of r and R using the multimeter and record these in your notebook. The circuit you are building is shown in figure 8.
- With CH1 of the ADC, measure the voltage drop on r, and then derive the current in the circuit, using $I = V_r/r$; measure the potential difference across R (V_R) with CH0 of the ADC.
- Use the program **CH1 vs. CH0** (Acquire mode) from the **4BL Main Menu** and set a sampling rate of 5,000 samples/sec and ensemble size of 1,000 points. Take one ensemble of data and save the file. When loading the data set into excel, the first column is time, the second column is CH0 voltage and the third is CH1 voltage.

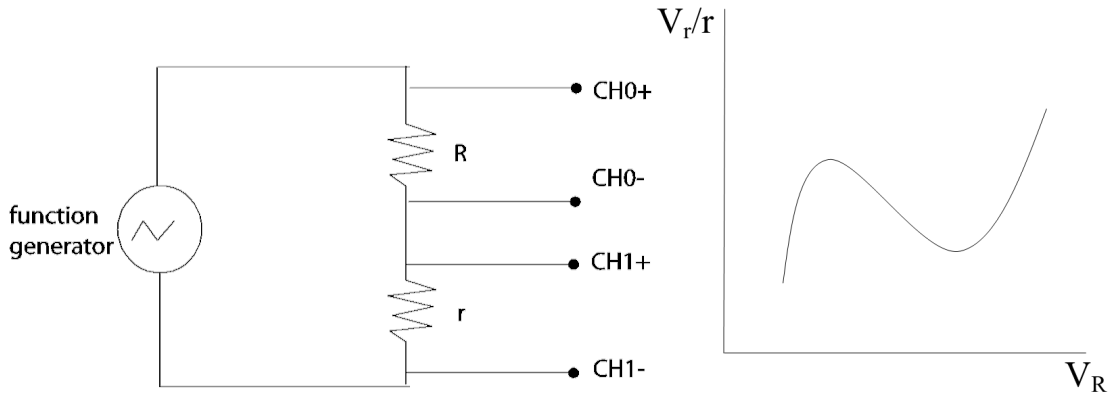

Figure 8: The circuit you will build to measure the IV curve of an unknown resistor R, verifying Ohm's Law. The voltage across a series resistor r of known value is used to determine the current.

Resistors in Series and Parallel Using the resistors at your lab station, you will form both a series and parallel combination, and test the equations for the equivalent resistance for the two cases.

- Pick 2 resistors from your lab station that have resistances somewhat similar to each other. First, use the digital multimeter to measure the resistance of each resistor individually (you will not be using the ADC for this part).
- Measure the equivalent resistance of the two resistors in series with the multimeter. Next, repeat the measurement for the parallel combination.

Compare your measured equivalent resistance of both the series and parallel combination with the calculated equivalent resistance based on your measured resistance of each individual resistor. In this situation, our measurement uncertainty is due by the precision of our measurement device. If our multimeter measures resistance to a tenth of an ohm, then you should use as your experimental uncertainty as $\delta R = 0.05\Omega$. See chapter 1 and 2 of Taylor for further discussion on estimating uncertainties. Is any discrepancy within your experimental uncertainty? Comment on this in your lab notebook (this portion will not be included in your report).

Deviation from Ohm's Law For this part, you will repeat the Ohm's law measurement, but replace the resistor R with a diode, and measure its $I-V$ relationship. Each lab station has a diode in your kit fit into a banana plug connector to implement into your circuit; it is black with a gray stripe. The circuit you will build for this part is shown in figure 9. Note that the gray stripe on the diode determines the direction of the diode: the gray stripe represents the vertical line at the tip of the arrow in the diagram representing the diode in figure 9. Repeat the steps from the Ohm's Law section to measure the IV curve of the diode. After performing the measurement, view your data in Excel by plotting CH0 voltage on the x-axis and CH1 voltage on the y-axis. If your data has discrete jumps as opposed to a continuous line, the resolution of the measurement is too coarse.

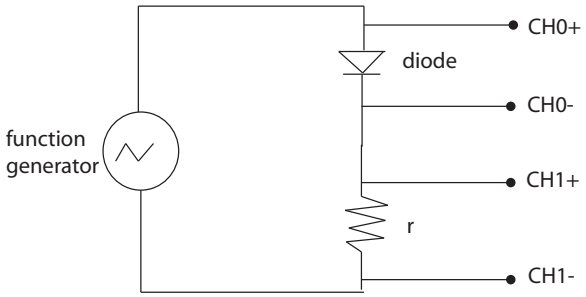

Figure 9: Circuit used to measure the IV curve of the diode. The function of a diode is dependent on the direction it is oriented in the circuit. In the diagram, this direction is defined by an arrow. The voltage across a series resistor r is used to determine the current.

The measurement should be repeated with increased resolution on CH1. This is accomplished by changing the CH1 limits from $\pm 10V$ to $\pm 5V$ or $\pm 0.5V$ in the CH1 vs. CH0 menu.

The Conductivity of Copper You are provided a coil of copper wire which is connected to a banana plug. You can use the banana plug to measure the resistance of the wire, however we are interested in the *resistivity*. You must determine the dimensions of the copper wire and use equation 12 to determine the resistivity.

- Measure the resistance of the coil of copper wire using the multimeter.
- The wire can be considered a cylinder of radius r and length ℓ. The information we have is that the wire gauge is #24 AWG (American Wire Gauge, diameter d=0.021"). To infer the length of the wire, determine the mass by weighing it. The mass density of copper is ρ_m=8.920g-cm^{-3}. By knowing the diameter, mass density, and weight, you can determine the length of the wire.
- From the *Handbook of Chemistry and Physics*, we have $\rho_{Cu}(T = 293K)$=$1.7\mu\Omega - cm$. Compare this value to your result, and comment in your lab notebook.
- Next, we want to cool the copper by immersing it in liquid nitrogen. Ask the TA to pour some liquid nitrogen into the styrofoam cup provided at your lab station. Cool the test piece of copper to LN_2 temperatures by slowly lowering the winding into the liquid nitrogen. Then, use the multimeter to measure the resistance.

Note that over a wide range of temperatures, we expect that the temperature dependence $\rho(T)$ of a metal such as copper is approximately given by

$$\rho(T) = \rho_0 + AT,$$

with A a constant that we associate with Cu. The constant ρ_0 varies with imperfections such as chemical impurities and disorder in the arrangement of copper atoms. In your notebook, record the change in resistance at the two temperatures and calculate the resistivity. (this portion will not be included in your report).

II.2.2 AC circuits - week 2

Transient state measurements First, you will build an RC circuit with a resistor and capacitor in series and measure the voltage on the capacitor during its charging. After that, you will build an RL circuit and measure the voltage on the inductor during the transient period. Instead of connecting the circuit to a DC battery as described in the indroductory section, we will use a square waveform produced by the function generator, which will repeatedly charge and discharge the capacitor or inductor. You will measure the voltages with the ADC of the computer and you will need to save the data to file to perform data analysis.

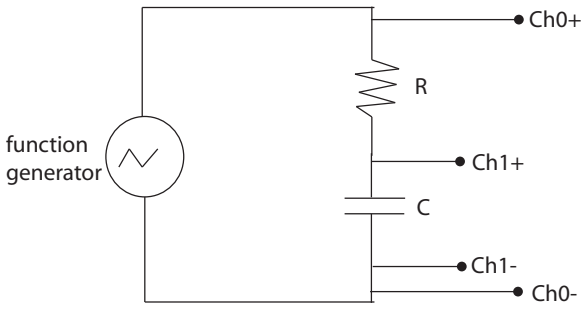

Figure 10: An RC circuit is connected to a function generator to measure the charging of the capacitor as a function of time.

- Choose a resistor and capacitor combination giving a time constant $\tau = RC$ in a convenient range to measure using the ADC in the computer ($\approx 10^{-3}$ seconds). Build a series connection circuit on top of the aluminum break-out box. The circuit you are building is shown in figure 10.
- Next, apply a square waveform with frequency $f < 1/(2\pi\tau)$. Set the amplitude of the waveform to be about 2V, and use an offset equal to the amplitude, so that the waveform is at zero volts during the lower portion of the square wave. The waveform you want to produce is shown in figure 11. Use the oscilloscope at your lab station to verify the form of the square wave. Once the desired waveform is programmed, connect the signal to the aluminum break out box.
- Use the "Acquire waveforms (2 channels)" module, use Ch0 of the data acquisition system to measure the input waveform (the voltage drop across the series combination of R and C), and use Ch1 to measure the voltage on the capacitor.
- Set the sample rate and the total points of measurement so that one or two full periods are recorded and visible on the display. Observe how the voltage on the capacitor changes when the input voltage jumps from zero volts to its peak voltage. Save the data file.
- Substitute the capacitor for an appropriate inductor L, by again choosing a combination which yields a time constant $\tau \approx 10^{-3}$ seconds.

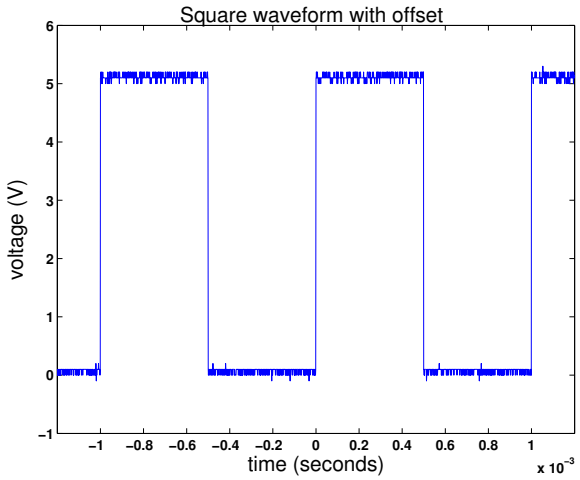

Figure 11: A square waveform is used as a impulse step-up voltage to study the transient of RC and RL circuits. The square wave is offset so that the lower voltage is at zero volts.

- For the RL circuit, we will need to build a slightly different circuit, including a parallel combination with a small resistor. Figure 12 shows the circuit you will construct. The reason for this is because the function generator has an internal resistance of 50Ω, which will be similar in magnitude to the resistor used in your RL circuit. By using a small resistor with $R \approx 5\Omega$ in parallel with our RL circuit, we are effectively reducing the input impedance, as seen by our RL branch.

- With Ch0 measuring the voltage drop across the RL combination and Ch1 measuring the voltage drop across only the inductor, repeat the measurement of the transient time period as you did for the RC circuit, and save the data to file. You may need to adjust the limits from $\pm 10V$ to $\pm 5V$ or $\pm 0.5V$ to get adequate resolution, so it is a good idea to plot your data in Excel before moving to the next session.

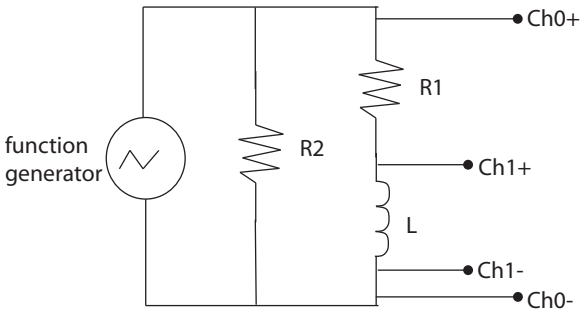

Figure 12: The RL circuit used to measure the transient period includes a resistor and inductor in series. A parallel combination with a small resistor effectively reduces the input impedance seen by the RL series combination.

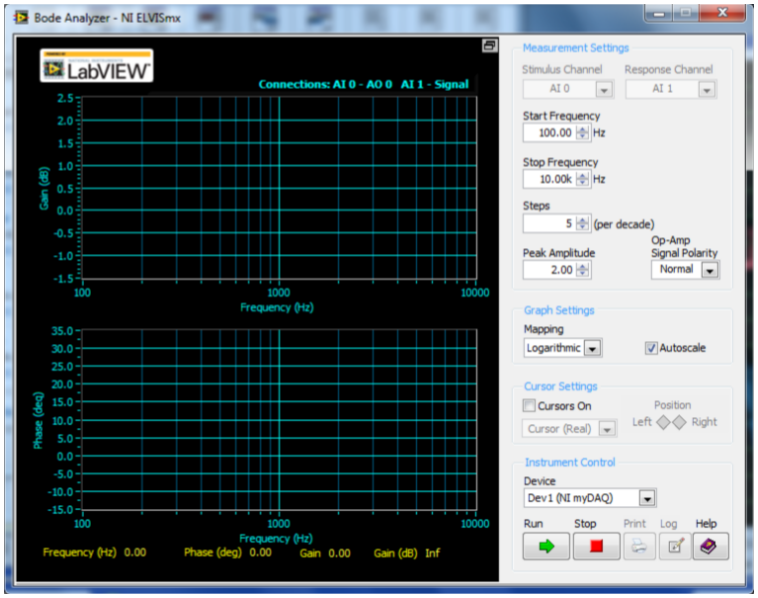

Figure 13:

Resonance Next, you will build a RLC resonant circuit, and measure the performance of the circuit as a function of frequency. For this section, you will use the BODE Analyzer in the myDAQ software. A screenshot of the BODE software interface is shown in Fig. 13. This tool is located in the Instruments panel on the windows toolbar. Select the BODE icon to start the software. The softwares purpose is to drive an AC current through a circuit and measure the voltage response. It graphs two quantities as a function of frequency, the voltage response gain (Eq. 19) and the phase (which we are not concerned with). The goal of this exercise is to find the resonant frequecny for our circuit and determine its "quality factor" Q.

- Connect the myDAQ channels as shown in Fig. **??**. Choose a $1\mu F$ capacitor, the $1k\Omega$ and use the wire coil used for the liquid nitrogen part of the DC lab as the inductor. Note that the power for the circuit comes from the white wire in Ch. A01 and is then grounded with the green wire. Be careful to make sure you are measuring the voltage properly over the correct elements with CH0 and CH1.
- In the software adjust the options as you see fit in order to get a graph which will give you a good view of the resonance peak. This could possibly include the start/stop frequency, the number of steps or the peak amplitude.
- Once you have a clean graph that shows the resonant peak, save the data by using the Log button at the bottom right hand side of the panel. Import this data into Excel to do your data analysis.

II.3 Analysis and Lab Report Guidelines

Last week, in the *Uncertainty and Statistics* assignment, you learned how to assign a best guess estimate to a single variable that you measured, and to assign an uncertainty value to that measurement. In the present assignment, you took data of two variables that had some functional form. For example, in the first part of the assignment, you are to verify Ohm's Law I=V/R by measuring

both the current I and the voltage V. This week you will learn an analysis method called "linear regression," which, assuming that the relation is linear, enables you to make a best guess at the function parameters and assign a uncertainty value. In the case of Ohm's Law, this means a linear regression is used to calculate R, and assign an uncertainty δR to that value.

Lets assume we make 10 measurements of two variables x and y: $(x_1, y_1), (x_2, y_2), ..., (x_{10}, y_{10})$. Additionally, we expect the two variables to vary linearly, with the form:

$$y = mx + b.$$

Performing a linear regression enables us to calculate the best guess at m and b. Since our measurements of x and y will inherently have uncertainty in their measured values, the linear regression will also enable us to quantify the uncertainty in m and b, due to this measurement uncertainty. To perform the analysis, we will assume that one of the variables y is the dominant source of error, meaning that our uncertainty in y is much larger than the uncertainty in x. Thus, we can neglect the uncertainty in x when performing our analysis (the case when the uncertainty in both variables is relevant is a bit more complicated).

The idea behind a linear regression is to find the line through the data points which minimizes the distance of the data points from that line. Mathematically, this means to find the m and b such that the quantity

$$\sum_i (y_i - mx_i - b)^2$$

is minimized. The quantity in the parentheses is the distance of what we measured for y_i from what we calculate y to be based on our measurement of x_i. The quantity is squared so that no cancelation occurs in the summation. Because of this, the method is also called a "least squares fit." Minimization of the above summed quantity with respect to the two fitting parameters yields our best fit parameters:

$$\hat{m} = \frac{N\sum xy - \sum x \sum y}{N\sum x^2 - (\sum x)^2} \tag{20}$$

Once the best guess of the slope is found, the intercept can by determined by:

$$\hat{b} = \frac{N\sum x^2 \sum y - \sum x \sum xy}{N\sum x^2 - (\sum x)^2} \tag{21}$$

For a derivation of these equations, see chapter 8 of Taylor. Now, since there was experimental uncertainty in measurements of the y_i's which we can denote by σ_y, we must assign some uncertainty in these best fit parameters m and b. Lets denote these uncertainties as σ_m and σ_b respectively. A simple application of error propagation shows that:

$$\sigma_m = \sigma_y \sqrt{\frac{N}{N\sum x^2 - (\sum x)^2}} \tag{22}$$

$$\sigma_b = \sigma_y \sqrt{\frac{N}{N\sum x^2 - (\sum x)^2}} \tag{23}$$

Then, when you present your values for m and b in a scientific paper such as your lab reports, you present them as $m = \hat{m} \pm \sigma_m$ and $b = \hat{b} \pm \sigma_b$. To analyze the data from this assignment,

you will perform a linear regression on the data sets of your various measurements. You can easily calculate the above equations in Excel, however Excel has built in analysis software that can do these calculations for you. Appendix C describes how to load the analysis software in Excel, and how to perform a linear regression. Note that the instructions are written for the older version of Excel (prior to Excel 2007).

II.3.1 Ohm's Law

To analyze your data from the Ohm's Law section, you will perform a linear regression to determine the value of the "unknown" resistor R, and also the uncertainty in this value. You have saved to file the data from your measurements to verify Ohm's Law. The data file contains three columns: column A is time (not used in your analysis), column B is Ch0 voltage (which is the voltage across R), and column C is Ch1 voltage (which is the voltage across r).

- First, make column D the current, by dividing the values in column C (the voltage on r) by the resistance of the "known" resistor r (which you measured with the multimeter and recorded in your lab notebook).
- Perform a linear regression, using column B (Ch0 voltage) as the x values, and the current (column D) as the y values. You can do this either by using the Analysis software built into Microsoft Excel, or by using the built in analysis tools of Excel to calculate the slope, and its uncertainty. For Ohm's Law, $I = V/R$, the intercept is zero. Because of this, equation 20 is simplified to:

$$\hat{m} = \frac{\sum xy}{\sum x^2}. \tag{24}$$

  and the uncertainty in the slope is simplified to:

$$\sigma_m = \frac{\sigma_y}{\sqrt{\sum x^2}} \tag{25}$$

- The output of the linear regression gives the slope 1/R. Invert this number to determine the resistance of the "unknown" resistor R, and compare with the value you measured with the multimeter.

II.3.2 Deviation from Ohm's Law

In the "Deviation from Ohm's law" section, you measured the IV relationship of a rectifier diode. The expected relationship is given by equation 11, which exhibits an exponential dependence. You will perform a linear regression on this data in order to determine the constant $\frac{|e|}{k_B}$. It is recommended that you do each step of the calculation in successive columns (Col A,B,C...) and that you carefully label what each column is throughout your process.

- You will be performing the analysis on the "forward biased" portion of the data only. The forward bias regime is when the applied voltage (which is measured by CH0) is positive. You can use Excel's sort function to group the data such that it is arranged by increasing CH0.

- Convert the voltage across the resistor r (CH1 voltage) into current by dividing by r, similar to what you did in the previous section. You want want to make a new column with this data.

- We start by determining the value of I_0. This is a constant of the diode under study, and is the value of the current when the applied voltage (CH0) $\rightarrow -\infty$. In your data, find the value of the current when the largest negative voltage is applied and use this value for I_0. Note that your value of I_0 is much smaller than the rest of your measured values for the current, that is $I_0 << I$. So for our calculations we can say that $I + I_0 \approx I$.

- In order to perform a linear regression, which requires the relationship to be linear, we must first "linearize" the data. The exponential I(V) relationship

$$(I + I_0) = I_0 \left(e^{|e|V/nk_BT} \right)$$

  can be linearized by taking the natural log of both sides and simplified using the approximation we noted before:

$$\ln(I) = \ln(I_0) + (\frac{e}{nk_BT})V \tag{26}$$

- Now you are ready to perform a linear regression on the data by plotting the voltage across the diode on the x-axis and plot $ln(I)$ on the y-axis. **Note:** The majority of your data may look very messy for small and negative values of V. In this region the exponential relationship is not strong and will disrupt the regression. You will need to cut off these data points until you only have the nice relationship. When you plot your regression line, it should fall right on top of your data. You can do the regression using the tools in Excel.

- The value of the slope determined by the regression is the ratio $\frac{e}{nk_BT}$. (The constant n depends on the diode's material and fabrication. Typically this value varies from 1 to 2. For the diode you will be using, the value is close to 2, so set n=2.) Multiply the slope value by nT= $2 \cdot 293$ K (room temperature measured in Kelvins) to determine the ratio e/k_B which are two fundamental constants of nature.

II.3.3 Transient Measurements

In the transient state measurements, you built and RC and an RL circuit and measured the voltage across the reactive component during the transient period. By performing a linear regression after the data is linearized, you can determine the characteristic time scales of the transient, $\tau = RC$ for an RC circuit and $\tau = L/R$ for an RL circuit. You are required to perform the analysis for only *one* of the circuits (either the RC or RL circuit), however you will need to present the raw data for both circuits (see the report guidelines below).

- Load the RC circuit data into Excel. Column A is time, column B is Ch0 voltage which is the input square wave voltage, and column C is the voltage across the capacitor. The input square wave voltage is either at zero volts, or at the peak voltage, which we can denote by V_b. In your Excel file, find the *row* where the input voltage (column B) jumps from zero to V_b, and lets call this row i. The input voltage will stay at V_b for multiple rows until it drops back to zero. Find the last row where the voltage is at V_b and lets call this row j. In column D, copy the *time* (column A) from rows i to j. In column E, copy the *Ch1 voltages* (column

C) from rows i to j. Columns D and E now contain the time and capacitor voltage for only the transient time period.

- Next, we are going to linearize the data as we did in the previous section, since the voltage on the capacitor has an exponential dependence (equation 15). In column F, calculate $V_b - V$ by subtracting column E from your value of V_b. In column G calculate $\ln(V - V_b)$ by taking the natural log of column F. Equation 15 has now been linearized:

$$\ln(V_b - V) = \ln V_b - \frac{1}{RC}t$$

- perform a linear regression with time (column D) as your x-values, and voltage (column G) as your y-values. The value of the slope given by the linear regression is 1/RC. Invert this number to determine τ and compare with the values R and C you used in the circuit.

- Load the RL circuit data into Excel. Column A is time, column B is the input square wave voltage, and column C is the voltage across the inductor. The input square wave voltage is either at zero volts, or at the peak voltage, which we can denote by V_b. In your Excel file, find the row where the input voltage (column B) jumps from zero to V_b, and lets call this row i. The input voltage will stay at V_b for multiple rows until it drops back to zero. Find the las row where the voltage is at V_b and lets call this row j. In column D, copy the *time* (column A) from rows i to j. In column E, copy the *Ch1 voltages* (column C) from rows i to j. Columns D and E now contain the time and inductor voltage for only the transient time period.

- Next, we are going to linearize the data, since the voltage on the inductor has an exponential dependence (equation 17). In column F calculate $\ln(V)$ by taking the natural log of column E. Equation 17 has now been linearized:

$$\ln V = \ln V_b - \frac{R}{L}t$$

- perform a linear regression with time (column D) as your x-values, and voltage (column F) as your y-values. The value of the slope given by the linear regression is R/L. Invert this number to determine τ and compare with the values R and L you used in the circuit.

II.3.4 Resonant Circuit

The final task of your assignment was to build a resonant RLC circuit, and measure its performance as a function of frequency. To do this, you measured the peak-to-peak amplitude of the output voltage at different frequencies. At the circuit's resonant frequency, the output voltage was maximized. For analysis, you will compare your measured and theoretical values of f_{res} and calculate a quantity called the quality factor, denoted by Q, which measures the sharpness of the resonance peak.

- Compare your measured value of f_{res} with the theoretical value $f_{res} = \dfrac{1}{2\pi\sqrt{LC}}$.

- calculate the circuit's Q-factor, which measures the width of the resonance peak and the power loss in the circuit. First, find the voltage V_{max} at f_{res}. Then calculate $V_{max}/\sqrt{2}$, and

draw a horizontal line of this level on your graph. This line will intercept with your resonance curve at two points, with frequencies f_1 and f_2 respectively. Finally, the Q value of the circuit is determined as

$$Q = f_{res}/(f_2 - f_1).$$

II.3.5 Lab report guidelines

Your lab report should contain the following sections

1. **Header**

   - Descriptive title
   - Name, Date, and class information
   - Name of your lab station partners

2. **Introduction**

   The introduction section explains, in your own words, the purpose of your experiment and how you will demonstrate this purpose. Try and be as brief as possible, yet still get your point across.

3. **Experimental Results**

   This section should briefly explain what was measured and how the data was collected (diagrams of the experimental set up may be helpful), present the raw data in graphical form (if possible) including uncertainty values with all numbers. Your graph should include labels on both axes that include units. Label your graphs as figure 1, figure 2, etc, so that you can refer to them in your text. In the text, explain the meaning of the variables on both axes and include an explanation that makes it clear to the reader how to interpret the information displayed on in the plots.

   your report should contain:

   - graph of IV data used to verify Ohm's Law. Explain the functional form.
   - graph of IV data for the diode. Explain the functional form.
   - graph of the RC transient (voltage on capacitor as a function of time). Explain the functional form.
   - graph of the RL transient (voltage on inductor as a function of time). Explain the functional form.
   - graph showing the output voltage of the RLC resonant circuit as a function of frequency. Explain the functional form.

4. **Analysis**

   this section should present any calculations performed on the raw data, including uncertainties using propagation of errors. Example: if you are testing Ohm's Law $V = IR$, and you measured V and R, then here you calculate I. If you have a theoretical value for your calculated quantity (I in this example), state whether it is within your uncertainty range. Also, answer any questions proposed in the lab manual.

   you should have:

- results of the linear regression on the Ohm's Law data, stating the resistance R and its uncertainty. Compare this R with the value measured by the multimeter.
- results of the linear regression of the diode, stating the estimate of e/k_b and its uncertainty. Compare with the accepted values for these fundamental constants.
- results of the linear regression for *either* the RC or RL circuit giving $\tau = RC$ for an RC circuit or $\tau = L/R$ for an RL circuit.
- Compare the measured and theoretical values of the resonant frequency and calculate the Q-factor of your resonant circuit.

5. **Conclusion**

   State how your results demonstrate (or fail to demonstrate) the objectives you presented in the introduction section.

III Laboratory 3: Magnetism

III.1 Background information

In this laboratory exercise you will make measurements of magnetic fields generated by steady state sources. By using a device called a Hall probe, you will measure the spatial dependence of the magnetic fields and verify the theoretical predictions. You will also investigate the coupling of magnetic and electric fields predicted by Faraday's Law using a time-varying source of magnetic fields. Not all measurements will be required in your report, so be sure to make thorough notes in your lab notebook to receive credit for completing these sections.

III.1.1 Magnetic fields produced by currents

The Biot-Savart law gives the magnetic field for any known current distribution. Ampère's law can also be used in situations of high symmetry. In the case of a long, straight wire carrying a current I directed along the z-axis, both calculations yield an azimuthal magnetic field of magnitude

$$B_\varphi = \frac{\mu_0 I}{2\pi\rho} \tag{27}$$

where $\mu_0 = 4\pi \times 10^{-7}$V-s/A-m is the permeability of free space and ρ is the radial distance from the line current.

In a real experiment an infinitely long line current cannot be created. All dc currents have to be closed and a second return-current carrying conductor is required. This can be accomplished with two coaxial cylindrical conductors, which preserves the axial symmetry. At the ends of the cylinders the current has to close radially. In the cylindrical gap between the conductors the field is the same as for an infinitely long line current on axis. Instead of passing a large current through a single solid conductor it is more practical to use a small current and make the conductors out of thin wires with a large number of turns. Thus a toroidal coil with N=100 turns has been built, and is shown in figure 14. In this section of the present assignment, you will use a Hall probe to measure the spatial dependence of the magnetic field produced by the toroid to verify equation 27.

The inner radius is a=3.2 cm, the outer radius is b=18 cm, the axial height is h=46cm. The wire spacing of about 1 cm allows one to insert the Hall probe radially, and a hole allows access to the center.

III.1.2 Magnetic fields produced by permanent magnets

In this section you will investigate the magnetic field produced by a permanent magnet. In this type of material the dipole moments associated with electronic spins all align in a common direction, producing a magnetization (dipole moment per unit volume) $\mathbf{M} \neq 0$. Let's consider the case where the magnetization is uniform throughout a volume V. Then the total dipole moment $\mathbf{m}$ in V is simply given by

$$\mathbf{m} = \int_V \mathbf{M} dV.$$

At distances $|\mathbf{r}|$ much larger than the linear dimensions of V, the magnetic field generated is very well approximated by that of a dipole moment $m = |\mathbf{m}|$ directed along the z-axis:

Figure 14: A toroidal coil produces a magnetic field identically to that of an infinitely long straight current. Making the conductors out of multiple turns of thin wire allows access to measure the field at all radial positions.

$$B_z = \frac{\mu_0}{4\pi}\frac{m}{r^3}(3\cos^2\theta - 1) \tag{28}$$

$$B_\rho = \frac{\mu_0}{4\pi}\frac{3m}{2r^3}\sin 2\theta \tag{29}$$

where θ is the angle measured from $\hat{z}$ (the polar angle), and $\hat{\rho}$ is the unit vector directed radially. The distance from the magnet geometric center is $r = \sqrt{\rho^2 + z^2}$. We will be measuring the magnetic fields at $\theta = 0, 90deg$ which leads to $B_\rho = 0$ so we will only measure z-components of the field.

A cylindrical ferrite magnet (2 cm diam, 2.5 cm long) which is axially magnetized is supplied for studying its properties. The north pole is indicated by a yellow dot. **B** is directed outward from a physical north pole (note that the geophysical north pole is a physical south pole). The magnet is mounted on a nonmagnetic circuit board and plugs vertically into the aluminum connector box used only for support. The magnet is shown in figure 15. Using the Hall probe, you will measure the spatial dependence of the field produced by the permanent magnet and verify the given equations.

III.1.3 Force between magnets

We have shown in the previous section that at distances far from a permanent magnet the field it produces decays as it would for a dipole moment. Then, the energy of interaction between two dipole moments $\mathbf{m_1}$, $\mathbf{m_2}$ is given by

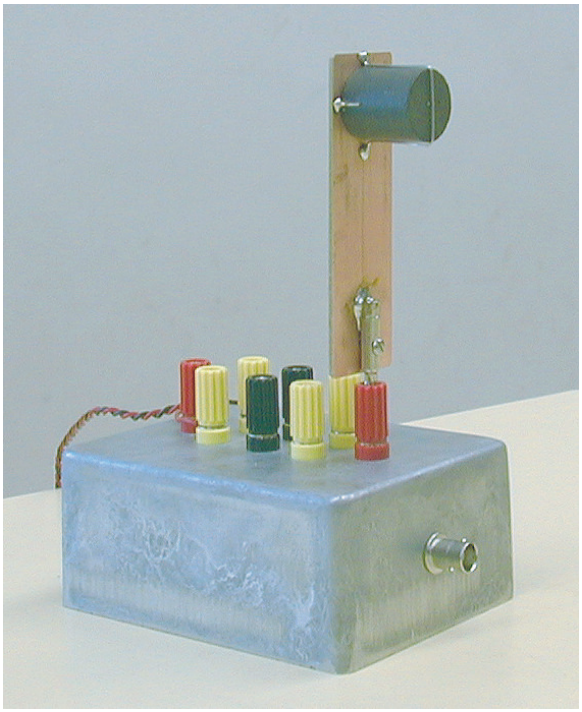

Figure 15: a permenant magnet produces a B-field which, at large distances as compared to the magnet size, can be approximated as a dipole field.

$$U = -\mathbf{m_1} \cdot \mathbf{B_2} \tag{30}$$

where $\mathbf{B_2}$ is the B-field produced by dipole $\mathbf{m_2}$. This field takes the form given in the previous section. The force between the two magnets is calculated using

$$\mathbf{F} = -\nabla U. \tag{31}$$

Since the magnetic field produced by a dipole varies as $B \propto r^{-3}$ (equations 28 and 29) the force between two dipoles should vary as $F \propto r^{-4}$. You will verify this spatial dependence of the force between two dipoles in the section of the present assignment.

III.1.4 Faraday's Law

One of the four Maxwell's equations is Faraday's Law, describing the fundamental effect of magnetic induction. In differential form, it reads

$$V_{ind} = -\frac{d\Phi}{dt} \tag{32}$$

where V_{ind} is the induced potential difference in a loop and Φ the magnetic flux through one loop. If your loop has N turns the induced voltage will be

$$V_{ind} = -N\frac{d\Phi}{dt}.$$

Here we will verify this expression experimentally. We use a small coil (N=100 turns, mounted in a plastic cylinder), into which we insert a known amount of magnetic flux. This is done by inserting a smaller solenoid (with n=8turns/mm and 2.2cm diameter) inside of the coil and driving a current through the inner solenoid. The current will vary sinusoidally:

$$I(t) = I_0 \sin(\omega t)$$

The strength of the magnetic field inside a solenoid is approximated as (since this assumes an infinite solenoid)

$$B = \mu_0 n I \tag{33}$$

where n is the number of turns per length and I is the current in the solenoid. The flux inside the larger coil will be changing in time due to the time-varying current through the smaller coil, and thus a voltage will be induced in the larger coil. You will measure this induced voltage, whose magnitude we can calculate using equations 32 and 33 and knowledge of the dimensions of the coils. The result is (verify this for yourself):

$$V_{ind} = I_0 \mu_0 \omega A n N \cos(\omega t) \tag{34}$$

where A is the cross sectional area of the smaller coil, I_0 is the amplitude of current in the smaller coil, n is the turns per length of the smaller coil, and N is the number of turns for the larger coil.

III.1.5 Magnetism in materials: paramagnetism and diamagnetism.

We have already worked with some permanent magnets in the previous section; the materials producing those fields are classified as ferromagnets, and the phenomenon is called ferromagnetism. Most materials do not retain a permanent magnetic moment producing a field. However, even where no permanent moment is observed, magnetic moments are induced when a material is placed into a magnetic field. Generally, we consider two possible responses: (1) The induced moment aligns with the magnetic field (paramagnetism). (2) The induced moment anti-aligns with the magnetic field (diamagnetism). All magnetism in matter is quantum mechanical. The distinction between ferromagnetism, paramagnetism, and diamagnetism lies in the origin: usually paramagnetism is produced by the coupling of electronic spin angular momenta to the magnetic field, and diamagnetism is associated with the coupling of electronic orbital angular momenta to the magnetic field. Ferromagnetism arises when there is an interaction between spins that otherwise would be paramagnetic.

In this section, we will examine a few sample materials for signatures of paramagnetism and diamagnetism. This will be done by measuring the sense of the force between a permanent magnet, and either a paramagnetic or diamagnetic test sample. In one case the force is attractive, and in the other it is repulsive. Let's start with a permanent magnet of total dipole moment $\mathbf{m}$, located at $\mathbf{r}$. When it is nearby to a sample S, the field $\mathbf{B_m}$ from $\mathbf{m}$ *induces* a moment $\mathbf{m_S}$ in S. Then, as before, the potential energy associated with this is given by

$$U = -\mathbf{m_S} \cdot \mathbf{B_m}.$$

For the paramagnet, $\mathbf{m_s} \parallel \mathbf{B_m}$, so U becomes more negative as $|\mathbf{B_m}|$ grows larger. Recall that the force will be in the direction for which U decreases most quickly. That is,

$$\mathbf{F} = -\nabla \cdot U,$$

and thus, the paramagnet is always attracted to the permanent magnet.

In weak fields, the induced moment of a paramagnet or a diamagnet is proportional to the magnetic field in which the material is placed; for this case, we have:

$$\mathbf{M} = \frac{1}{\mu_0}\chi\mathbf{B},$$

where χ=constant is the magnetic susceptibility. You will be provided a set of materials and it is your task to determine which are diamagnetic and which are paramagnetic.

III.1.6 Experimental Setup

To measure magnetic field strength, you will use a Hall probe Gaussmeter. The Gaussmeter (F. W. Bell, Model 5080) consists of two components, (i) a Hall probe and (ii), an electronic instrument with power supply and electronics to convert the analog Hall voltage into a calibrated digital readout. The device is shown in figure 16.The principle of a Hall probe is the following: A dc current flows through a semiconductor Hall element in the x-direction. When a magnetic field is applied in the y-direction a charge separation occurs and produces a voltage drop along the z-direction. This Hall voltage is proportional to B and is calibrated in Gauss.

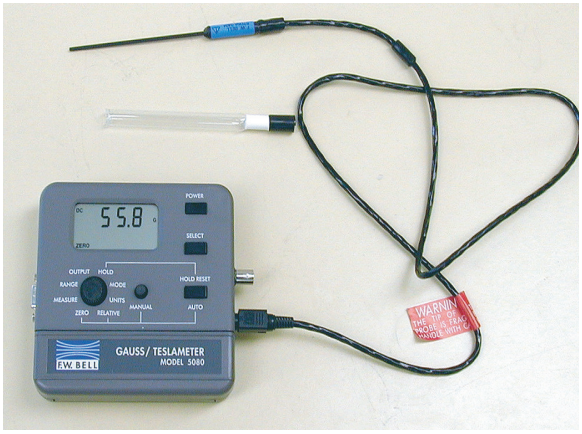

Figure 16: The Gaussmeter consists of a Hall probe (thin filament) and the digital electronics. Be careful in that the Hall Probe is a fragile and expensive device.

The use of the Gaussmeter is straightforward but requires one important knowledge: Caution! The Hall probe is very fragile. Do not touch or bend the probe tip, which contains the sensor element. Just hold the probe in the vicinity of current- carrying wires or magnets without touching them. Note that the Gaussmeter probe is a transverse probe, which measures the field component normal to its axis and perpendicular to its flat side. Rotation around the probe shaft will give a sinusoidal variation of the readout. The Hall element is located at the tip of the probe. Become familiar with the properties of the instrument by reading the instruction manual, especially the warning signs.

Before taking a measurement, place the Hall probe far from any sources of magnetic fields and zero the probe to remove the contribution due to the Earth's magnetic field. To do this, rotate the knob to the "zero" position and press the "Reset" button. When the readout settles on a null measurement, return the knob to the "measure" position. Some measurements require measuring the field along one direction. This is accomplished by mounting the probe on a linear track, which is shown in figure 17. The probe is held with alligator clips on a small stand bolted to the movable

table of the track. Clamp one alligator clip onto the round shaft covered in blue tape (DO NOT CLAMP THE FILAMENT ITSELF!), the other on the flexible cable covered by black tape. Adjust the stand so as to measure the desired field component in the desired direction.

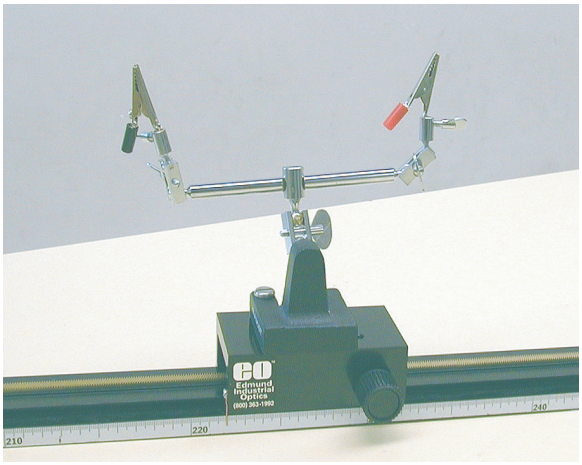

Figure 17: A linear track is used to incrementally move the Hall probe along a direction. The Hall probe is attached to a linear track by clipping the probe with alligator clips. Clamp one alligator clip onto the round shaft covered in blue tape (DO NOT CLAMP THE FILAMENT ITSELF!)

III.2 Procedure and Measurements:

III.2.1 Magnetic fields produced by line currents

- Apply around 15-20V, to the coil from the dc power supply. If the power supply overloads (indicated by the red light turning on and the voltage supplied going to zero), increase the current limit and/or use a smaller voltage $\approx 10V$ is fine.

- Verify that the magnetic field between the axial conductors is in the azimuthal direction as predicted by the current flow. To do this, carefully hold the probe in the region between the two conductors by hand. Rotate the Hall probe to measure the different components of the magnetic field, remembering that the Hall probe only measures the component of B perpendicular to the flat surface of the filament. Which direction do you need to orient the Hall probe to get a nonzero reading?

- Verify that at a constant radius ρ the field does not depend on azimuthal position. To do this, simply move the Hall probe around the toroid, inserting and at different positions but at the same radius. Keep the probe oriented to measure the azimuthal component of the field. Next, verify the field does not depend on height. To do this, insert the probe in the intermediate region and move the Hall probe vertically, again keeping it at a constant radius and measuring the azimuthal component. Given these facts, you can prove that the measured field and current satisfy Ampere's law,

$$\oint B \cdot dl = 2\pi\rho B_\varphi = \mu_0 I_{enclosed} = \mu_0 NI \tag{35}$$

- Now measure the radial dependence $B_\varphi(r)$. Note that inside the inner conductor ($r < a$) the field vanishes since no current is enclosed. Between the conductors (a < r < b) verify that $B \sim r^{-1}$, and outside the outer conductors (r > b) the field vanishes since there is no net enclosed current. To do this, align the linear track so that the Hall probe moves along the radial direction. Note that there is a hole in the black plastic cylinder of the toroid. This is provided so that the Hall probe can reach the inner region of the toroid (which is inside the inner copper wires). Be sure to align the linear track so that the Hall probe can enter this hole as you take your radial measurements. Measure the azimuthal field at about 20 to 25 positions of equal spacing (every 0.5cm is suggested), starting inside the inner conductor and finishing outside the toroid. The ruler attached to the side of the linear track can be used to measure *changes in position*. At home, you'll need to convert this distance into the actual radial distance, which you can do with the knowledge that the inner conductor is at a radius of 3.2cm. Record the position and field strength in your notebook.

- After completing your measurements, measure the voltage applied to the coil and the resistance of the coil with the multimeter to determine the current.

III.2.2 Magnetic fields produced by permanent magnets

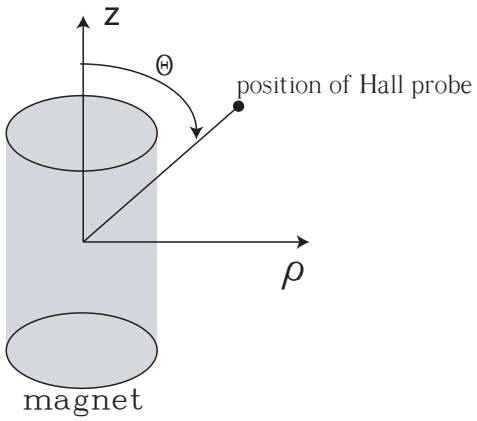

Figure 18: orientation of the permanent magnet. The z-axis is defined along the symmetry axis of the magnet.

You will measure the magnetic field produced by the permanent magnet, and verify the field profile is that of a magnetic dipole. The z-axis is defined along the symmetry axis of the cylindrical magnet as shown in figure 18. You will measure the spatial dependence for two cases: $\theta = 0$ degrees and $\theta = 90$ degrees. For the two cases, you are to verify the field is in the correct direction and that the field strength decreases with distance as described by equations 28 and 29.

- measure $\mathbf{B_z}(\Theta = 0°, z)$, i.e. along the z-axis. To do this, align the Hall probe on the linear track so that the motion of the probe is along the z-axis as defined above, and so that the

Hall probe is measuring the z-component of the field. Measure the field strength at 10-20 distances starting a few cm away from the top surface of the magnet to verify the field dependence $B_z(z) \sim r^{-3}$. The ruler on the side of the linear track can be used to measure changes in position. Measure the starting distance between the probe and magnet with the ruler provided, and use this as the offset position. You can change the sensitivity of the probe from Guass to kGuass if needed.

- measure $\mathbf{B_z}(\Theta = 90°, \rho)$ by aligning the linear track so that the Hall probe moves in the radial direction and measures the z component of B, as defined in the figure. Start with the Hall probe a few cm away from the side of the magnet (again measure this offset position with your ruler), and measure the field strength for 10-20 positions along the radial direction. **Note:** It is really important that you begin your measurements several cm away from the edge of the magnet! Data too close to the magnet will not follow the radial dependence.

Compare your results to expectations based on Eq. 28, and 29.

III.2.3 Force between magnets

Next, you will measure the force between two permanent magnets as the distance of their separation is varied to verify the functional form. The force can be measured by placing one of the magnets on a scale. The readings of a scale can be converted from mass to force using the acceleration due to Earth's gravity, however this will not be necessary as we are only verifying the functional form as opposed to the exact value.

Figure 19: The setup to measure the force between two permanent magnets as the distance separating them is varied.

- Use two identical permanent magnets (2 cm diam, 2.5 cm long). Place one of them on a digital scale and null out its weight. The second magnet is attached to a circuit board, which you can mount into the linear track with the alligator clips. Clamp the linear track in the sturdy vise so that the magnet can move in the vertical direction. Note that there is a screw in the linear track which, when tightened, prevents the track from moving. The setup is shown in figure 19.
- Align the magnets axially with equal poles facing each other so that they repel. Notice that the weight increases. Do not exceed the weight limit of the scale (200 grams maximum.). Vary the dipole separation for 10-20 different positions and measure the force on the magnet, which can be done by converting the reading of grams into force. Measure the offset distance, which can be either the closest or farthest separation, with a ruler.

III.2.4 Magnetism in materials: paramagnetism and diamagnetism.

The setup is as follows: Clamp a stainless steel rod into the vise. Attach the nylon string through the slit at the upper end of the rod and make a few extra turns to adjust the length of the torsional pendulum. The height should be such that the diode laser beam strikes the mirror above the magnet. The magnet should be freely rotating above the table. Place the copper conductor about 2cm from and parallel to the broad side of the magnet to dampen the oscillations. Place a screen such that the reflected laser beam is in its center. Keep all iron and permanent magnets far away from the sensitive measurement device. Refer to figure 20.

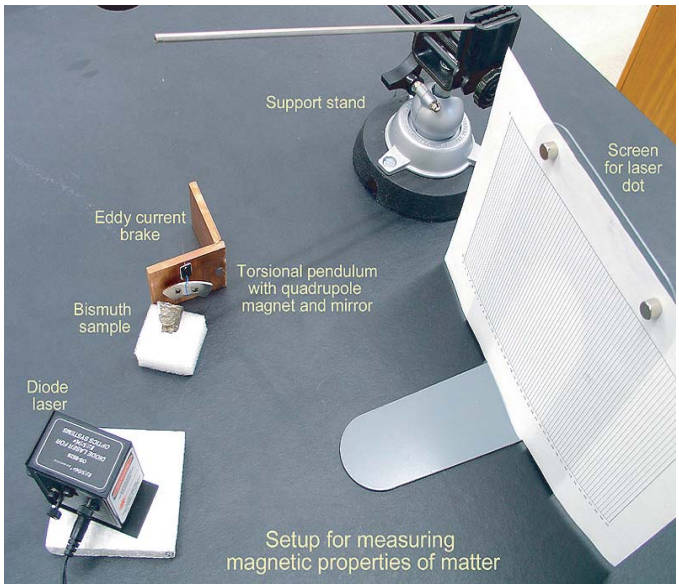

Figure 20: Setup for determining whether a material is paramagnetic or diamagnetic.

After the equilibrium position has been established place a magnetic material on a Styrofoam support and move it slowly to about 0.5cm from one of the magnet poles. Observe the rotation of the magnet, which is greatly magnified by the motion of the laser spot. If magnet and matter attract the latter is paramagnetic, if they repel it indicates diamagnetism. You are supplied with a variety of magnetic materials (Cu, Al, Ta, Bi, C, Fe, Ni, glass, rocks, etc).

Determine whether they are paramagnetic or diamagnetic. And record the results in your lab

notebook. You will not be including this section in your lab report so comment thoroughly in your notebook to receive credit. The setup is very sensitive and you can measure the diamagnetism of your finger! (why you are diamagnetic?). Please handle the delicate setup with great care: Do not touch the quadrupole magnet with iron objects or other magnets, do not touch the mirror surface with your hands, do not pull hard on the nylon string (2lb. fishing line). And of course, never look straight into the laser beam.

III.2.5 Faraday's Law

In this section, you will verify the expression for Faraday's law as explained in the introduction section. To do so, you will drive a time varying current through the inner coil and measure the induced voltage in the outer coil. You will measure both the current through the inner coil and the induced voltage in the outer coil with the myDAQ.

- Insert the solenoid into the Faraday coil and apply a $\sim 1kHz$ sine wave to it. You will also need to place a 10Ω resistor in series with the solenoid to measure the current going through the solenoid.
- Measure the current with CH0 (by measuring the voltage across the 10Ω resistor) and the induced voltage in the outer coil on CH1. Set the rate to 100,000 and number of points to 1000. Verify that the amplitude of the sinusoidal voltages you measure are what you expect based on Faraday's law, equation 34.

III.3 Analysis and Report Guidelines

In the previous laboratory exercise, you learned how to perform a linear regression on a set of experimental data. You measured N pairs of measured quantities x and y (x_1, y_1),...,(x_N, y_N) and assumed they were linearly related: $y_i = mx_i + b$. The linear regression analysis then provided you with best-guess estimates of the two fitting parameters m and b, as well as the uncertainty in these best-guesses σ_m and σ_b.

In the present assignment, you will learn an additional feature of the linear regression analysis called the *correlation coefficient*. The correlation coefficient, denoted by r, tells you how *well* the variables x and y are linearly related. The value of r ranges from 0 to 1, and the closer r is to 1 the better x and y are linearly related. Thus, if all your data points fit exactly on the linear fit, the r value will be vary close to 1. Conversely, if the data points deviate significantly from the linear fit, the r value will be close to zero. From your measured values of x and y, the correlation coefficient is calculated as:

$$r = \frac{\sum(x_i - \overline{x})(y_i - \overline{y}}{\sqrt{\sum(x_i - \overline{x})^2 \sum(y_i - \overline{y})^2}} \tag{36}$$

where $\overline{x}$ and $\overline{y}$ are the average values of the N x_i and y_i. For further details on this equation, see chapter 9 of Taylor.

For the present laboratory assignment, you are asked to determine how well your experimental data agrees with theoretical predictions. For example, in the first section, you measure the magnetic field produced by a line current at different radial positions and are to compare with the theoretical prediction that $B \propto r^{-1}$. To do this, you will linearize your data, perform a linear regression,

and then see how close the correlation coefficient is to 1. Note that there is a systematic way to determine how close r must be to 1 in order to claim the two variables are linearly related, and you can refer to Taylor chapter 9 for details. However for this class, simply comment on the magnitude of r to interpret the strength of your linear fit.

III.3.1 Magnetic fields produced by currents

In this section, you measured the magnetic field produced by a toroid in three regions: inside the inner coil $r < a$, in between the two coils $a < r < b$, and outside the outer coil $r > b$. For the middle region, we expect the field strength to be related to the position by equation 27. To verify the 1/r dependence, calculate the correlation coefficient, which can be calculated by equation 36, or done in excel by performing a linear regression. To do so, insert your values for the B-field strength in column A of an Excel file, and the radial position of the measurement in column B. Next, linearize the data by calculating 1/r in column C (i.e. divide 1 by the values in column B). Then, performing a linear regression with column C as the x-values and column A as the y-values will evaluate the equation

$$B(x) = \frac{\mu_0 I}{2\pi} x$$

where $x = 1/r$. In the linear regression output, the square of the correlation coefficient r^2 is given. Comment on the correlation coefficient of the linear regression as to whether the magnetic field does in fact vary as 1/r for the middle region of the toroid.

III.3.2 Magnetic fields produced by permanent magnets

In this section, you measured the magnetic field produced by a permanent magnet along two directions of symmetry: along the polar z axis and along the radial direction. Equations 28, and 29 state that the field strength should vary as $1/r^3$. You are to calculate the correlation coefficient for *one* of the directions. As in the previous section, this can be done in Excel. First, linearize the data by calculating $x = 1/r^3$. Then, perform a linear regression with this quantity x as your x-values and the field strength as your y-values. Use the r^2 value given by excel to comment on the agreement between your measured values and theory.

III.3.3 Force between magnets

In this section, you measured the force between two dipoles. The introduction section demonstrates that the force should depend on the separation distance as $F \propto 1/r^4$. Calculate the correlation coefficient (a step-by-step explanation isn't needed at this point) to comment on the agreement between your measured values and this theoretical prediction.

III.3.4 Faraday's Law

In this section, you measured the voltage induced in a coil of wire due to a time varying flux through the coil. The time varying flux was created by driving a sinusoidal current through a solenoid which was placed inside the coil. You also measured this current, by placing a resistor in series with the solenoid and measuring the voltage drop across this resistor. Loading your data file in Excel gives time in Column A, the voltage across this resistor in column B, and the voltage induced on to coil in column C. In column D, calculate the current by dividing column B by the resistance of the

series resistor. Plotting column D on the y-axis and column A on the x-axis should display the current varying sinusoidally with an amplitude I_0, which you should determine. Plotting column C on the y-axis and column A on the x-axis should display the voltage induced on the coil varying sinusoidally with an amplitude V_0. Compare this measured value of V_0 with that predicted by Faraday's law (eq: 34) using your measured value for I_0.

III.3.5 Lab report guidelines

Your lab report should contain the following sections

1. **Header**
   - Descriptive title
   - Name, Date, and class information
   - Name of your lab station partners
2. **Introduction**

   The introduction section explains, in your own words, the purpose of your experiment and how you will demonstrate this purpose. Try and be as brief as possible, yet still get your point across.
3. **Experimental Results**

   This section should briefly explain what was measured and how the data was collected (diagrams of the experimental set up may be helpful), present the raw data in graphical form (if possible) including uncertainty values with all numbers. Your graph should include labels on both axes that include units. Label your graphs as figure 1, figure 2, etc, so that you can refer to them in your text. In the text, explain the meaning of the variables on both axes and include an explanation that makes it clear to the reader how to interpret the information displayed on in the plots.

   your report should contain:

   - graph of Field strength vs radial distance for the field produced by a toroid, showing all 3 regions. Comment on the magnitude of the field for the inner $r < a$ and outer $r > b$ regions and the vector nature of the field inside the toroid $a < r < b$.
   - graphs of field strength vs distance for the field produced by a permanent magnet (two orientations). Comment on the vector direction of the field based on your measurements.
   - graph of force between two permanent magnets vs separation distance.
   - graph of current in the inner coil and induced voltage in the outer coil as a function of time.
4. **Analysis**

   this section should present any calculations performed on the raw data, including uncertainties using propagation of errors. Example: if you are testing Ohm's Law $V = IR$, and you measured V and R, then here you calculate I. If you have a theoretical value for your calculated quantity (I in this example), state whether it is within your uncertainty range. Also, answer any questions proposed in the lab manual.

   you should have:

- Graph of field strength on the vertical axis and 1/r on the horizontal axis for the field due to a line current. Perform a linear regression to determine the correlation coefficient and comment in the results.
- Graph of field strength on the vertical axis and $1/r^3$ on the horizontal axis for the field due to a permanent magnet (for only one direction: along z or ρ). Perform a linear regression to determine the correlation coefficient and comment in the results.
- Graph of force on the vertical axis and $1/r^4$ on the horizontal axis for the force between dipoles. Perform a linear regression to determine the correlation coefficient and comment in the results.
- Compare the measured and theoretical values of the voltage induced on the Faraday coil.

5. **Conclusion**

   State how your results demonstrate (or fail to demonstrate) the objectives you presented in the introduction section.

IV Laboratory 4: Speed of Sound and Light

IV.1 Background Information

In this laboratory exercise, we will measure the speed of two most commonly encountered waves: sound waves and electromagnetic waves. A speaker will be used to detect sound waves and a microwave antenna, called a "Yagi" antenna, is used to detect electromagnetic waves in the microwaves regime. You will also investigate the angular dependence of the antenna's performance. You will have 1 laboratory session to perform the measurements, and your report will be due the following week.

IV.1.1 Sound waves

Sound waves propagate in matter with speeds depending on the mode of propagation and properties of the medium. For instance, only longitudinal waves propagate through gases and liquids, and the speed is typically much less in gases. Solids will carry transverse waves, as well as longitudinal. In our own experience, we are most familiar with sound in air; human hearing is sensitive to waves of characteristic frequencies covering the approximate range f=20Hz-20kHz. In science and technology, we use devices called transducers (microphones and speakers) to excite/detect non-audible sound for research and applications (sonars in oceonography or submarine warfare, seismic waves in geophysics, echograms in medicine, etc). Thus, the knowledge of sound waves is important in many areas of modern life.

Plane monochromatic waves are described by the expression

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi),$$

where $\mathbf{k}$ is the wavevector, $\omega = 2\pi f$ is the angular frequency, and $\mathbf{r}$, t are the spatial and time coordinates. The direction of propagation is determined by $\mathbf{k}$; arbitrarily defining it to be along the x-axis results in

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}\cos(k_x x - \omega t + \phi),$$

where $\mathbf{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ and $k_y = k_z$=0. Note that the polarization of the wave is determined by $\mathbf{A}$.

We quantify the rate at which a wave is traveling with the phase velocity. This is the rate at which wave's phase propagates through space. The phase velocity is calculated as:

$$v_p = f\lambda$$

where f is the frequency of the wave and λ is the wavelength. In terms of the angular frequency ω and the wavevector k, this expression becomes

$$v_p = \frac{\omega}{k}.$$

By measuring the frequency and wavelength of a sound wave, you will be able to determine the phase velocity of sound traveling in air.

Sound waves traveling through air are also non-dispersive. This means that all frequencies travel at the same velocity. A prism is an example of a dispersive medium: when light travels through a prism, the different frequencies (and thus the different colors) that make up white light travel at different velocities. This is what enables you to see the spectrum of visible light (red to violet)from a white light source. You will demonstrate that sound waves traveling through air are non-dispersive by measuring the velocity of sound for different frequencies.

IV.1.2 Electromagnetic waves

Electromagnetic waves propagate in free space (vacuum) at the speed of light, $c = 3x10^8$m/s. They can also propagate through some forms of matter, but at a reduced speed. c is one of the most fundamental constants in physics: relativity theory postulates that no energy or matter propagates faster than the speed of light. Astronomy depends on the knowledge of the speed of light to estimate the size of the universe. Communication via electromagnetic waves is limited by the speed of light, *e.g.*, the speed of computers or the interaction with satellites in space.

The speed of light also connects the most fundamental physical quantities of space and time. These are defined rather than derived. Historically, space or distance is defined by comparison with geophysical quantities, for example the length of the Earth's equator. Time is derived from periodic events, initially also geophysical phenomena such as a day, month, year, etc. Later it was replaced by oscillating systems of increasing precision, first a pendulum in a mechanical clock, then a quartz oscillator in a digital clock, and now an atomic transition in cesium known to 15 digits accuracy. By subdividing the high oscillation frequency into fractions, long time intervals are measured with the same accuracy. Knowing time or frequency with such precision, the constant speed of light allows one to determine distance with the same precision. Increased precision of fundamental constants is an important effort in science since it can lead to new discoveries not detectable with poorer resolution.

Visible light is just a narrow spectrum of electromagnetic waves. In order to measure the propagation speed we will use microwaves with wavelengths that are easily measurable. The speed is calculated from $c = \lambda f$, and the frequency is set by the radiation source, a 2.45GHz cordless telephone. As in the case of sound waves we generate standing waves with a movable wall and measure the spacing between two amplitude maxima or minima. We use a directional dipole antenna to receive the electromagnetic waves. The high-frequency signal is rectified with a microwave diode and produces a dc voltage proportional to the wave amplitude.

Although the measurement principle appears the same as for sound waves there are fundamental differences between the two types of waves. The sound wave is a longitudinal wave in a neutral gas involving no electric or magnetic phenomena (except for the exciter and receiver). Electromagnetic waves are transverse waves which exist in the absence of matter.

IV.2 Experimental Setup

In this laboratory sound waves are produced by a planar bi-directional loudspeaker based upon the piezoelectric effect, shown in figure 21. Such flat speakers are used in cell phones, laptop computers and small radios. The sound wave is detected with a microphone that produces an electrical signal proportional to the sound wave. The microphone is shown in figure 21. The principle of the microphone is that of a parallel plate capacitor with one electrode being a flexible membrane; the circuitry produces an output signal with time-dependent potential that depends on the capacitance changes. The microphone is nearly omnidirectional.

Electromagnetic waves can be excited by oscillating currents in wire antennas and induce oscillating voltages in similar receiving antennas. Dipole antennas transmit/receive optimally for wave propagation perpendicular to the dipole axis. For maximum transmission the transmitting and receiving dipoles should therefore be parallel to one another. Variations of dipole antennas can produce directionality. You will produce electromagnetic waves using a cordless phone, which transmits in the microwave regime at 2.45GHz ($= 2.45 \times 10^9$Hz). The phone is shown in figure 22. Note that the polarization of the emitted elecromagnetic waves is determined by the orientation of the

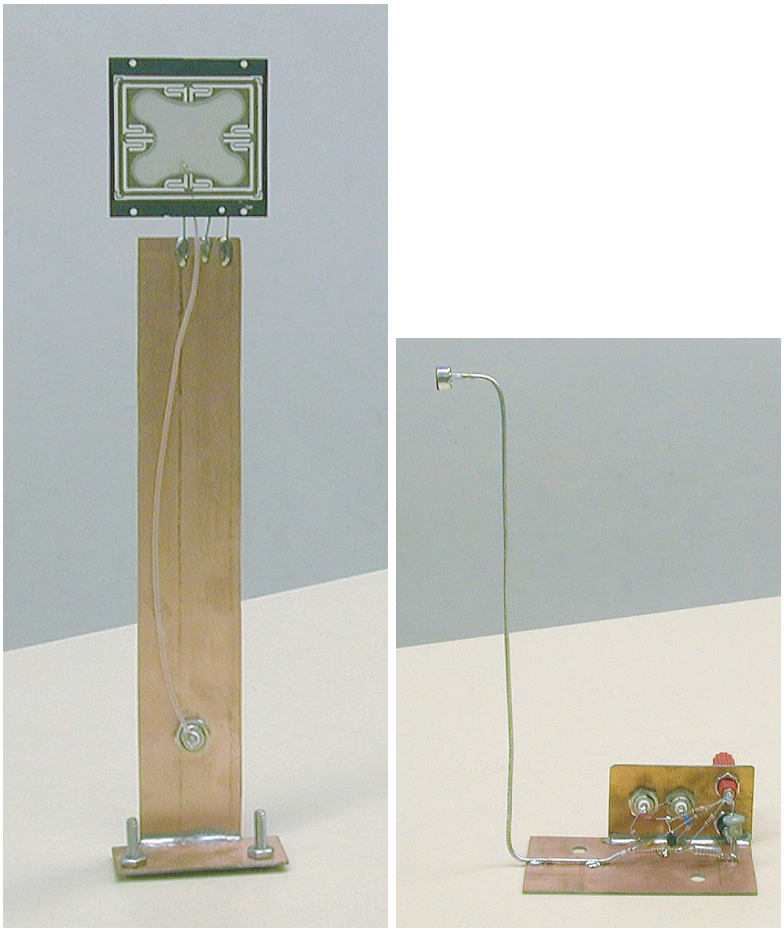

Figure 21: Speaker used to produce sound waves and microphone used to detect them.

phone's antenna. Also, be sure to turn the phone on when you want to produce EM waves. Your receiving antenna is a directional "Yagi" antenna similar to old analog TV antennas and is shown in figure 22. The antenna consists of a half-wavelength dipole, which is the middle element, and two parallel parasitic wires on either side. The shorter wire is called the "director" and the longer is called a "reflector" wire. The dipole wire is the wire that detects the electromagnetic waves. The two parasitic wires partially cancel or amplify the signal to produce directionality, meaning that the antenna receives signals from one direction better than another.

IV.3 Procedure and measurements

IV.3.1 Traveling sound waves

The setup for this part is shown in figure 23 and contains two elements: the loudspeaker which produces sound waves, and the microphone to detect them. The microphone and its amplifier have three connections: the first is a dc voltage of V_{dc}=12V, provided by a small power supply that is to be connected to the wall socket. There are two BNC connections, one supplying the AC signal measured by the microphone, and the other a DC signal proportional to the measured amplitude. The DC signal is the middle BNC. The microphone and its support are mounted on a movable track with a ruler attached so as to measure the waveform at different distances from the speaker.

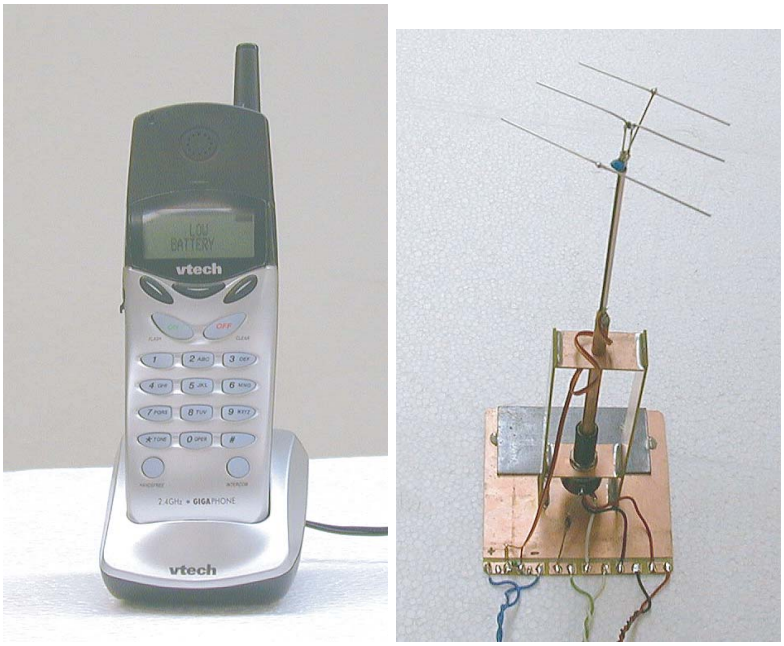

Figure 22: Electromagnetic waves are produced by a 2.45GHz cordless phone and are detected with a Yagi antenna.

The speaker has a BNC connection, which is to be connected to the function generator to drive the speaker.

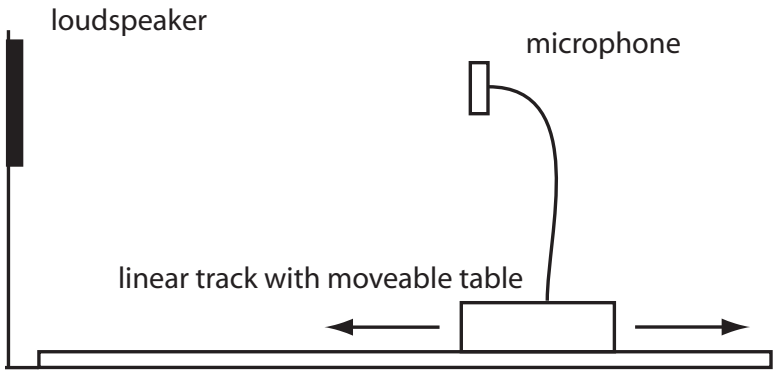

Figure 23: Schematic diagram of the laboratory setup for variation of the phase shift of the sound wave detected by the microphone.

- Connect the speaker to the function generator and apply a sine wave in the frequency range 2kHz-20kHz. Use the BNC T-connector to split the signal from the function generator so you can send it to the speaker and also to the oscilloscope. Have CH1 of the oscilloscope display the function generator signal (trigger off of it) and CH2 display the signal received by the microphone. Use the AC BNC of the microphone for this part. Observe on the oscilloscope the waveform of the sound signal versus time at a fixed position z, $V_0(t) = \cos(\omega t - \phi)$, where V_0 is the amplitude, ω the angular frequency, and ϕ an arbitrary phase shift.

- Now, we will very the position of the microphone and observe the phase shift of the signal detected by the microphone with respect to the signal driving the speaker. Move the microphone away from the speaker and observe that the phase shift with respect to the driving signal increases linearly with distance, $\delta\phi = k\delta z$; k=constant is the wavenumber. The distance over which the phase changes by 2π is the wavelength, so that $2\pi = k\lambda$.
- Measure the wavelength of the sound waves by measuring the distance the microphone moves when the phase changes by π and 2π, and record the distances in your notebook. Also record the frequency of the driving signal using the oscilloscope measurement. You can then determine the phase velocity of a sound wave in air, $V_p = f\lambda$. Be sure to determine the uncertainty in your distance and frequency measurements to perform error propagation.
- Repeat this measurement for about 5 different frequencies in the range 2-20 kHz so that you can plot the dispersion relation, f vs. k. This will show that the sound speed does not vary with frequency, $v_p = f\lambda$ is a constant.

IV.3.2 Standing Sound Waves

In this part of the lab, we will create standing waves, as opposed to traveling waves in the last section. The superposition of oppositely propagating, equal amplitude waves forms a standing wave pattern,

$$V_0 \cos(k_x x - \omega t) + V_0 \cos(k_x x + \omega t) = 2V_0 \cos(k_x x) cos(\omega t).$$

In a standing wave the phase does not shift but the amplitude $2V_0 \cos(k_x x)$ varies sinusoidally with position. Amplitude nulls (nodes) occur at spacings

$$k_x x = n\frac{\pi}{\lambda},$$

n=integer. Thus, the node spacing is one half wavelength. The same holds for the spacings of the amplitude maxima. If the incident and reflected wave amplitudes have *different* amplitudes their superposition yields maxima $(V_1 + V_2)$ and minima $(V_1 - V_2)$, each spaced one half wavelength apart.

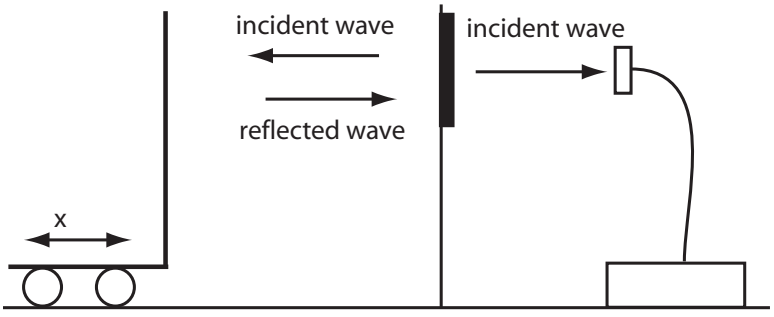

Figure 24: Schematic diagram of the setup for measuring the wavelength of standing waves.

To create the wave traveling in the opposite direction, we will use a reflector as depicted in figure 24. The reflector is mounted on wheels and moving its position will cause the standing wave pattern to move as well. By measuring the nodes and antinodes in the microphone signal as a function of the reflector position, you can determine the wavelength of the sound signal.

- The position of the reflector can be measured electrically because one of its wheels is connected to a linear 10-turn potentiometer. When looking at the shield from the back, you will see four banana plug connections. The left two leads are the power supply inputs to the potentiometer, which you need to connect to the power supply at 2V. The right two are the output signal of the potentiometer, which acts as a voltage divider, outputting a fraction of the input voltage depending on the position of the wheel.

- Calibrating the shield position: The potentiometer voltage needs to be calibrated by measuring the actual distance the shield moves with the ruler. Thus, measure the voltage produced by the potentiometer output with the multimeter as you move the shield at fixed distance increments (5cm for example) with respect to the ruler. Plotting the voltage as a function of distance will give you the voltage-to-centimeter calibration conversion. This calibration will remain valid as long as you don't change the voltage input. Keep the voltage on so that you don't need to recalibrate when you work with the microwave standing waves.

- Measure the standing waves: the position voltage and the DC signal from the microphone will be measured by the the myDAQ simultaneously. Use the program "CH1 vs CH0"; feed the position voltage into CH0 and the DC signal from the microphone into CH1 (using the aluminum connector box to convert from the BNC connection to banana plug connection). Under "measurement type," select *continuous*. Set the sampling rate to 0.1s. Set the frequency of the sinusoidal function generator output to f=5kHz and connect this signal to both the sound speaker and the oscilloscope (to measure the frequency) by using the BNC T-connector. Once you start the data acquisition, move the reflector slowly, keeping the microphone at a fixed position. By converting the position voltage into a distance using your calibration, you can measure the distance between either maxima or minima to determine the wavelength and thus the speed of sound. Try this measurement a couple of times and view the data in Excel by plotting CH0 on the x-axis and CH1 on the y-axis. You should see an sinusoidal signal with at least 4 or 5 oscillations. Note that this measurement does not require a display of the phase with an oscilloscope.

IV.3.3 Antenna properties

You will produce light waves using a cordless phone, which transmits in the microwave regime of 2.40 - 2.45GHz. Your receiving antenna is a directional "Yagi" antenna similar to old analog TV antennas. The antenna consists of a half-wavelength dipole, which is the middle element, and two parallel parasitic wires on either side, as depicted in figure 25. The shorter wire is called the "director" and the longer is called a "reflector" wire. The wires connect to a DC signal proportional to the amplitude of the electromagnetic wave detected by the Yagi antenna. This type of antenna exhibits directionality along the array axis, which you will observe in this section. A single dipole has no such directionality. Directionality is best understood by measuring the antenna radiation pattern, *i.e.*, the received signal as a function of angle between the antenna axis and the direction of wave propagation. **In free space the radiation pattern for exciting and receiving waves is the same (reciprocity).**

- Observing the antenna angle: For a fixed distance between exciter (phone) and receiver (antenna) (*e.g.*, 20cm), measure the received signal vs. θ of the receiving antenna. The excitation antenna of the phone should be parallel to the receiving antenna, and slabs of styrofoam are provided to elevate the phone. Measure the voltage change for one full rotation

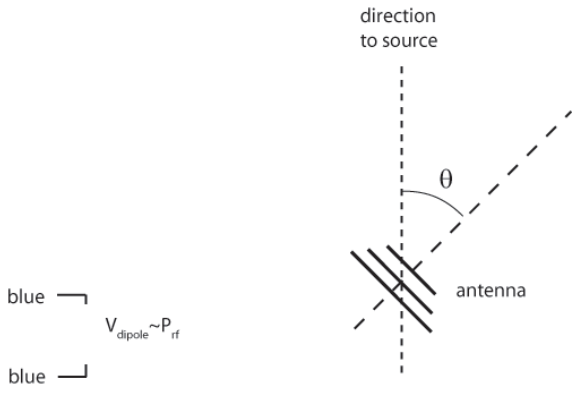

Figure 25: Connection diagram for the antenna.

of the Yaqi antenna and note the results in your lab notebook. You can hold the antenna base in place on the table and carefully rotate the central rod of the antenna. The microwave signal produced by the phone is rectified at the antenna with a Schottky barrier diode; the output V_{dc} will be quadratic in amplitude for small signals, which makes it proportional to microwave power. You should see a large signal at $\theta = 0$, a smaller peak at $\theta = \pi$ and zero signal for all other angles.

IV.3.4 Standing electromagnetic waves

Next, you will create an electromagnetic standing wave by radiating the 2.4 – 2.45GHz wave against a reflecting plane as shown in figure 26. As in the sound wave experiment, the reflector is a copper-clad circuit board, mounted vertically on a horizontally movable table.

- Place the receiving antenna between the reflector and the transmitter. Receiver and transmitter are at fixed positions (about 10 cm apart). Maximize the sensitivity of the antenna by orienting appropriately according to your results from Section IV.3.3. The transmitted e-m wave will be reflected and the superposition of incident and reflected wave forms a standing wave. The receiving antenna measures the field of the standing wave. When the reflecting plane moves away/toward the exciter the position of amplitude maxima and minima sweep past the fixed receiver antenna, and you can measure the wavelength of the microwave signal by measuring the distance between these nodes.

Figure 26: Schematic of the hardware positions for the detection of electromagnetic standing waves at f=2.45GHz.

- As in the previous section, use the program "Sample two Channels (continuously)"; feed the position signal into CH0, the DC signal from the Yagi antenna (blue wires) into the CH1, and slowly move the screen through a few maximums while recording data. Measure the spacing between adjacent maxima or minima as before using the potentiometer calibration. This gives you the wavelength of the electromagnetic waves. Since the frequency is fixed at 2.45GHz, the wave speed can be calculated and compared with the accepted value of c=speed of light. Plot your data in Excel to verify you have recorded 4 or 5 oscillations.

IV.4 Analysis and Report Guidelines

IV.4.1 Traveling sound waves

The goal of this section is to measure the dispersion relation for sound waves traveling through air. This is done by measuring the frequency of oscillation for different wavevectors and plotting $\omega(k)$. For a non-dispersive medium, the frequency is simply linearly related to the wavevector:

$$\omega(k) = v_g k.$$

The slope of the relation is the phase velocity, i.e. the speed at which sound travels through air.Thus, by plotting $\omega(k)$ you can show that sound traveling through air is nondispersive, and determine the phase velocity.

- You have measured the wavelength of traveling sound waves for 5 different frequencies. Convert frequency f to angular frequency ω by using $\omega = 2\pi f$ and wavelength λ to wavevector k using $k = 2\pi/\lambda$.
- Plot the dispersion relation with the wavevector on the x-axis and angular frequency on the y-axis. Perform a linear regression with wavevector as your x-values and angular frequency as your y-values, making sure you require the intercept to be zero. The regression output gives you the slope of this relation, which is the phase velocity, and gives you the uncertainty in this number. Does your calculated value for the speed of sound agree with the accepted value within your experimental uncertainty?

IV.4.2 Standing sound waves

This section provides an additional way to measure the wavelength of sound waves by creating a standing wave pattern. By positioning the microphone at various positions throughout the standing wave pattern, you measured the position of subsequent nodes and antinodes. The position was recorded by measuring the voltage of a potentiometer which can be converted to a position using your voltage-meters conversion.

- first, determine your voltage-position conversion by plotting your calibration data. Put the voltage readings on the x-axis and the position measurements on the y-axis. The slope of this line gives you the conversion in units of distance/voltage.
- Next, open your data file for standing sound waves in Excel. CH0 of your data recorded the position voltage from the potentiometer and CH1 recorded the amplitude of the sound wave detected by the microphone. Convert the position data into distance by using your voltage-position conversion. Then, plot the microphone signal vs position, by putting the distance values on the x-axis and the microphone signal on the y-axis.

- Your plot should display five or six maxima and minima of the oscillation pattern. The distance between adjacent maxima or minima is equal to *one half* of the wavelength. Calculate the average value for the wavelength from these 5 or 6 values, and also calculate the standard deviation of the mean (see laboratory assignment 1).
- From your measurement of wavelength, determine the phase velocity of the sound waves, and calculate the uncertainty of that value using error propagation. Is the accepted value for the speed of sound within your experimental uncertainty? Which method of measuring the speed of sound was more accurate, and which had better resolution?

IV.4.3 antenna properties

Antenna design is a large engineering discipline and directionality is one feature considered in such designs. In this section you learned how to use a Yagi antenna and measured its directionality. This was done by rotating the antenna with respect to the source and measuring the amplitude of the signal detected by the antenna.

IV.4.4 Standing electromagnetic waves

This section provides way to measure the wavelength of electromagnetic waves by creating a standing wave pattern. By positioning the antenna at various positions throughout the standing wave pattern, you measured the position of subsequent nodes and antinodes analogous to the standing sound wave measurement. The position was recorded by measuring the voltage of a potentiometer which can be converted to a position using your voltage-meters conversion.

- Repeat the analysis from the standing sound waves section, this time using your data from the standing electromagnetic waves section. As before, calculate the average wavelength and the standard deviation of the mean for the measurement. Calculate the speed of light by multiplying by the source frequency (2.4 GHz) and determine its uncertainty using error propagation.
- Is the accepted value for the speed of light within your experimental uncertainty?
- Knowing the speed of sound waves and electromagnetic waves, calculate the time delay between seeing lightning strike and hearing the thunder, assuming the lighting strikes exactly 1km away from you.

IV.4.5 Lab report guidelines

Your lab report should contain the following sections

1. **Header**
    - Descriptive title
    - Name, Date, and class information
    - Name of your lab station partners

2. **Introduction**

   The introduction section explains, in your own words, the purpose of your experiment and how you will demonstrate this purpose. Try and be as brief as possible, yet still get your point across.

3. **Experimental Results**

   This section should briefly explain what was measured and how the data was collected (diagrams of the experimental set up may be helpful), present the raw data in graphical form (if possible) including uncertainty values with all numbers. Your graph should include labels on both axes that include units. Label your graphs as figure 1, figure 2, etc, so that you can refer to them in your text. In the text, explain the meaning of the variables on both axes and include an explanation that makes it clear to the reader how to interpret the information displayed on in the plots.

   your report should contain:

   - describe the experimental setup, and include a graph of the dispersion relation for traveling sound waves.
   - describe the experimental setup, and include a table containing the values measured for the wavelength of sound from the standing wave section.
   - Table containing the values measured for the wavelength of electromagnetic waves.

4. **Analysis**

   this section should present any calculations performed on the raw data, including uncertainties using propagation of errors. Example: if you are testing Ohm's Law $V = IR$, and you measured V and R, then here you calculate I. If you have a theoretical value for your calculated quantity (I in this example), state whether it is within your uncertainty range. Also, answer any questions proposed in the lab manual.

   you should have:

   - Results of the linear regression, determining the phase velocity of sound and its uncertainty.
   - determine the average wavelength and its uncertainty, from the standing sound waves. Calculate the speed of sound and the uncertainty in this calculated quantity.
   - determine the average wavelength and its uncertainty, from the standing electromagnetic waves waves. Calculate the speed of light and the uncertainty in this calculated quantity.

5. **Conclusion**

   State how your results demonstrate (or fail to demonstrate) the objectives you presented in the introduction section.

V Laboratory 5: Geometric Optics

V.1 Background Information

In the following experiments, you will investigate some of the geometric and physical properties of light rays. In particular, you will verify Snell's Law of refraction, and work with thick and thin lenses used to redirect and focus light rays. Parallel rays of white light are produced by a "ray box", and a collimated monochromatic beam is produced by a laser. Observations can be recorded using a digital camera. You will have one laboratory session to complete the measurements, and your report explaining your results will be due the following week.

Warning: You will use a diode laser in some of these experiments. Although it is only a low-power laser, **NEVER LOOK DIRECTLY INTO A LASER BEAM!** Permanent eye damage (burned spot on retina) may occur from exposure to the direct or reflected laser beam. The beam can be viewed without any concern when it is scattered from a diffused surface such as a piece of paper. The laser beam is completely harmless to any piece of clothing or to any part of the body except to the eye.

Keep your head at all times well above beam height to avoid accidental exposure to your own or your fellow students' laser beams. Do not insert any reflective surface into the laser beam except as directed in the instructions or authorized by your teaching assistant. Only have the laser on when performing an experiment, and be aware of the direction of the laser so as not to harm individuals at neighboring lab stations. The laser contains a high voltage power supply. Caution must be used if an opening is found in the case to avoid contacting the high voltage. Report any problems to your TA.

V.1.1 Snell's Law: refraction and total internal reflection

The index of refraction n in any material is defined by the speed of light in the material c',

$$c' = \frac{c}{n},$$

where c is the speed of light in vacuum. To a very good approximation n=1 in gases such as air, whereas in transparent solids, n is typically larger but still of order unity. For plastic or glass, n varies over the range 1.3-1.8.

Figure 27 shows a ray incident upon an interface between two different transparent materials. The materials have indices of refraction n_1 and n_2, respectively. If the incident ray first passes through a material with index of refraction n_i at an angle θ_i from normal incidence, then the *reflected* ray makes angle θ_r, with $\theta_i = \theta_r$. **Snell's Law** gives the angle made by the *transmitted* light θ_t, which travels into a material with index of refraction n_t:

$$n_i \sin\theta_i = n_t \sin\theta_t.$$

In this laboratory exercise, you will verify these relations between incident, reflected, and transmitted angles. Additionally, you can determine the index of refraction for the material your prism is made out of by applying Snell's Law.

An interesting consequence occurs when $n_i > n_t$. At some incident angle $\theta_1 = \theta_c < 90^{\cdot}$, the transmitted angle $\theta_2 = 90^{\cdot}$. When $\theta_i > \theta_c$ there is no transmitted light ray, and all of the light ray

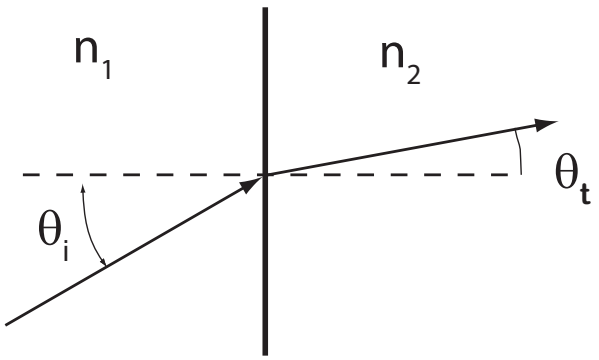

Figure 27: Behavior of a light ray at the boundary between two transparent materials. Light ray arriving from upper left in material type 1 gets reflected to upper right, and refracted into material type 2 at lower right according to Snell's Law.

is reflected. θ_c is called the critical angle, found from

$$\sin\theta_c = \frac{n_t}{n_i}.$$

Total internal reflection is the principle behind the transmission of light in fiber optic networks.

V.1.2 Lenses

In this section, you will study the properties of lenses. Your previous theoretical treatment of lens behavior assumed the limit of thin lenses, however in the first part you will be using thick clear plastic lenses to easily observe the path of the light rays.

You will first measure the defining property of a lens: its focal length, which is a measure of how strongly the optical device converges or diverges light. Secondly, we will see how these thick lenses deviate from the thin lens approximation by observing spherical aberration. Finally, you will use a combination of lenses to produce magnification and demagnification and calculate the magnification factor. We start with a series of parallel rays that enter a set of lenses. You will look at the separation between rays and use this to calculate the magnification factor. The magnification factor is defined as

$$M = \frac{\text{final separation}}{\text{initial separation}}$$

After being studying these properties using thick lenses, you will then switch to using *thin* lenses, which should agree well with the theoretical predictions from your lecture class. The focusing properties of a thin lens are expressed by a single quantity, the focal length f or its inverse, the diopters, $D = 1/f$, measured in units of m^{-1}. When two thin lenses are placed next to each other in a sequence they form a combined lens with focal length

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2}$$

or equivalently

$$D_{total} = D_1 + D_2.$$

When the two lenses have a finite separation $D = e$ the expression changes to

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{e}{f_1 f_2}.$$

You will verify this expression by measuring the focal length of two thin lenses individually, and then their combination.

Lenses are used to create an image of an object which is either magnified or demagnified. Consider an illuminated object at an axial distance o from a thin lens. A light ray from the top of the object passing through the center of the lens will not be deflected forming a straight line through the lens. A light ray from the top of the object and parallel to the axis will be deflected and pass through the focal point on the opposite side of the lens. An image is created where the two beams intersect. The distance i from the lens to the image plane, the focal length f, and the object length o are related by a simple expression,

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

(you can derive this result!). You will verify this expression by illuminating an object at a specific distance from a thin lens, and then measuring the distance from the lens to where the image is formed.

V.2 Procedure and measurements

V.2.1 Snell's Law and total internal reflection

A ray box will be used in this part to produce one or more rays of white light. These can be made to emerge from the box parallel to, or convergent/divergent from one another. The rays are incident on various optical elements (lenses and a prism) in order to observe how they are redirected on entering, passing through, and exiting optical objects.

After you provide power to the ray box, place an unlined piece of white paper under the ray box so you can more easily observe the rays. Arrange the filter so that 5 rays appear. By sliding the outer hull of the ray box back and forth, one can produce diverging, converging and parallel rays. Take a minute to make the rays as parallel as possible. *Be careful not to move it for the remainder of the experiment.* Periodically check to be safe. Note that the ray box gets hot very fast, so please take care in handling it!

- **Snell's Law:** Arrange the baffle (see figure 28) on the ray box so a single ray appears. On top of a white piece of paper, place the trapezoid prism so that the ray passes through undeflected. Then rotate the prism roughly (precision is not important!) 45° as in figure 29. Notice that there are actually two interfaces where the laws of ray optics apply: the front interface and the back interface. The light ray from the ray box is incident on the front surface, is reflected off of the front surface, and is also transmitted through the front surface inside the prism. You will be measuring the angles of these three rays to verify the laws of ray optics. These laws could also be verified for the back interface, by realizing the ray inside the prism, which is the *transmitted* ray according to the front interface, is now the *incident* ray with respect to the back surface.

  You will be measuring the angles by tracing the outline the prism, and tracing the location of the light rays, and then using a protractor to measure the angles.

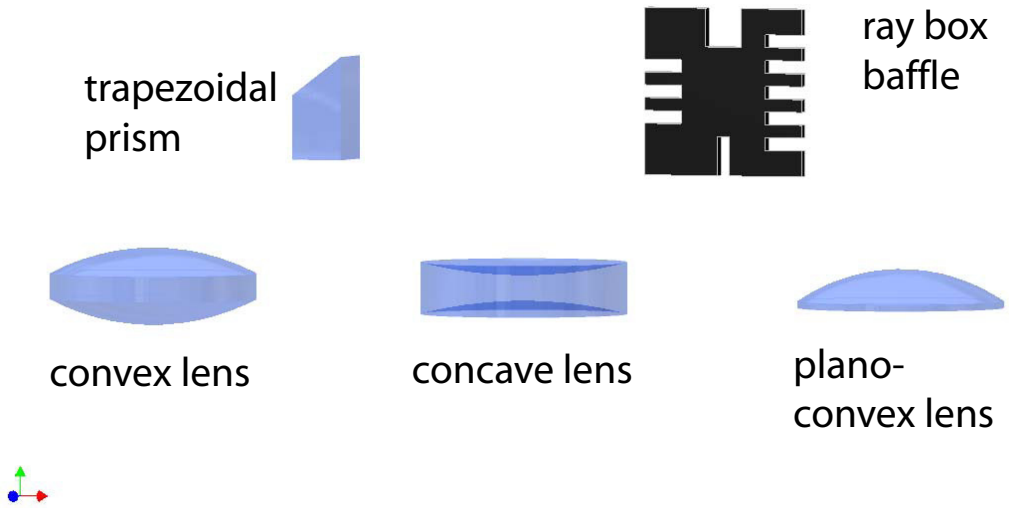

Figure 28: Components used with the ray box: lenses, prism, and baffle.

Trace the outline of the trapezoid once you are happy with its orientation, and be careful not to move anything while performing this section.

Trace out the location of all the light rays: the ray incident on the front surface, the ray reflected off of the front surface, and the ray transmitted through the second surface. Be sure to mark the position the rays enter and exit the prism so you can re-create the ray that travels through the prism.

To analyze, draw in the lines *normal* to each face of the prism where the ray entered/exited. Using a protractor, measure the incident and reflected angles for the front interface. Now measure the angle of the ray transmitted through the front interface (inside the prism). Use your measured angles, and the fact that $n_{air} = 1$ to calculate the index of refraction of the plastic by plugging into Snell's Law.

Next, use Snell's Law and the n you just determined to *predict* the angle at which the ray exits the *back* interface. Does this angle agree with the angle you measured?

- **Total internal reflection (TIR):** Your goal here is to observe total internal reflection. TIR will only occur on the back interface, since at this interface n_i is that for the prism, and $n_t = n_{air}$ satisfying the criterion $n_i > n_t$.

  Still using a single ray, arrange the trapezoid as follows: Allow the ray to pass through the slanted surface of the prism (the diagonally cut surface which, if excluded, would make the prism a rectangle). Slowly rotate the prism until the ray transmitted through the *back* interface disappears totally. This is total internal reflection. Trace the prism, the ray incident on the front interface, and mark the spot on the back interface where there is total internal reflection so you can draw the inner ray later. **Note:** Make sure you are measuring your angles at the precise orientation when the outgoing just disappears. It is easy to mistake a random orientation that has TIR; here we are looking only for the **critical angle**.

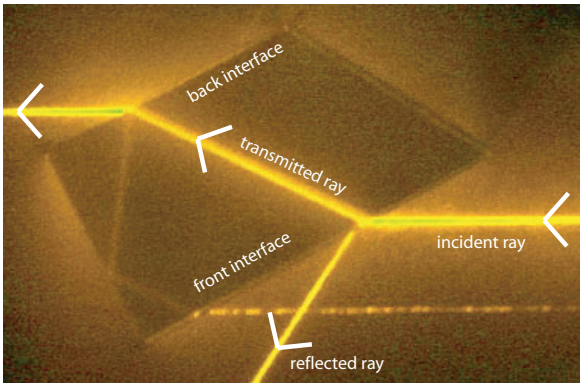

Figure 29: the trapezoidal prism is rotated approximately 45 degrees with respect to the incoming (from the left) light ray. The light ray reflects off of, and is transmitted through the front interface. Additionally, the light ray is incident on, and transmits through the back surface. By measuring the angles of incidence and transmission, Snell's Law can be verified.

As before, draw in the lines normal to the faces and measure the incident angle for the front interface, the transmitted angle for the front interface, and the incident angle on the back interface. The angle of incidence on the back interface is the critical angle for TIR. Using trigonometry and your value for the index of refraction determined above, find the critical angle and compare it to the value predicted by Snell's law.

V.2.2 Thick Lenses

- **Focal Length:** Arrange the filter so that 3 rays pass out of the ray box. Place the bi-convex lens (looks like a football) in front of the rays, with the axis of the lens normal to the rays. Observe that the 3 rays all converge to a single point on the far side of the lens. Now move the lens closer and farther away from the ray box, keep its axis normal to the rays, does the focal length change? With a new sheet of white paper, trace the lens and place a few dots along the rays to trace their path and measure the focal point. Be careful not to move anything while marking the paper. Repeat the same procedure for the biconcave and plano-convex lens. You will have to trace diverging rays backwards to the reverse side of the lens to find the focal point and thus its focal length.

- **Spherical Abberation:** Place the bi-convex lens back in front of the ray box and now allow 5 rays to fall on it, again making sure the lens axis is normal to the rays. Is there one focal point now or are there two? What do you think is happening? Try blocking out the middle three rays to see which rays converge to which focal point. This is called spherical aberration. Measure the distance between these two focal points and the distance from the lens to the midpoint between the two focal points. Then calculate the % difference between the two focal point positions, relative to the focal point of the central rays. Chromatic aberration is another lens problem which arises from the fact that the refractive index depends on color.

- **One Dimensional Magnification:** Allow 3 rays to pass out of the ray box, making sure the rays are parallel again, and place the biconcave lens near to the ray box. You should observe the diverging rays. If you now place the plano-convex lens just right, you can bring the diverging rays parallel once again. But now the rays should be farther apart! The original

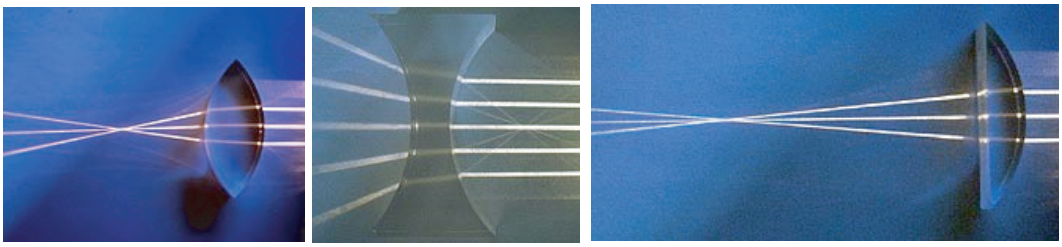

Figure 30: Convergence and divergence of light rays from three different lenses: Convex, Concave and plano-convex respectively.

spatial separation has been magnified. Trace the rays and measure the initial and final separation distances for the incoming and outgoing rays. Create the largest magnification you can. Repeat the above steps with the lenses placed in reverse order to produce demagnification. Then find the magnifications for the enlarging as well as reducing lens combinations.

V.2.3 Thin-lens Properties

The previous experiments dealt with thick, two-dimensional lenses that are rarely used. Now we will perform basic optics experiments with *thin* lenses used in most optical devices (cameras, microscopes, telescopes, glasses, etc). We will study the basic laws of lens combinations and image formation.

- First measure the focal length of individual lenses. For this purpose we generate two parallel (collimated) laser light beams. How is this done? We shine the single diode laser beam at an angle θ against a transparent plate, called a beam splitter. There will be two reflected beams, one from the front surface, the second from the back surface. The beams will be separated by a distance $d = D[sin(2\theta)](n_2 - sin^2\theta)^{-1/2}$, where n is the refractive index and D the plate thickness. Place the plate so that $\theta \approx 45^\circ$ on the optical bench, and shine the laser rotated by 90° to the bench against it to generate two parallel laser beams with separation $d \approx$1cm along the axis of the magnetic bench (see the figure). Take a few minutes to get the laser beams to travel parallel down the linear track.

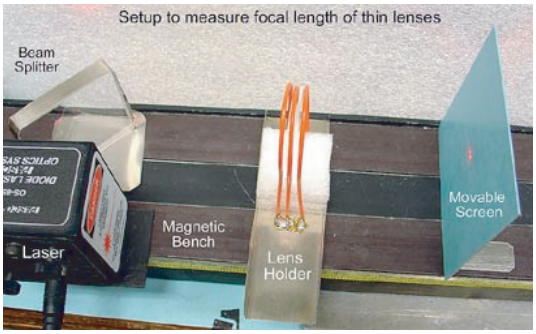

Figure 31: the setup for measuring the focal length for thin lenses. Parallel rays are produced by a beam splitter, and are then sent through an optical lens. The focal length can be found by observing the convergence of the two beams on a moveable screen.

- Shine the two parallel light beams through the middle portion of a thin lens held by the orange stand on the bench at an arbitrary distance from the laser. When you handle lenses always touch them at the edges but not in the middle, otherwise they will become dirty. If so, clean them with liquid soap and warm water and soft cloth or paper. Place the blue screen behind the lens and observe that with increasing distance from the lens the two light spots first converge to a single spot (the focus) and then diverge again. **Note:** When doing this measurement, take note of where the two laser beams start to overlap and where they stop overlapping. This will not be at the same point! Use these locations to approximate your error for this measurement. Using the metal ruler, measure the distance from the center of the lens to the focus and calculate $D = 1/f$. Use a different lens and repeat the measurement. Place the two lenses together and measure D_{total}. Check whether your result agrees with the prediction.

- Also measure the focal length of the 2cm diam lens supported by a vertical twisted wire which will be used in the next experiment on image formation.

- Furthermore measure the focal length of a "thick" lens, an acrylic sphere. For such a ball lens the focal distance from the *center* is given by $f = nD/[4(n-1)]$ where n is the refractive index (n=1.5) and D=25.4 mm is the diameter of the sphere. Measure the focal length from the surface, $f' = f - D/2$, and compare your measurement to the theory. Ball lenses are used to couple fiber optics cables.

V.2.4 Image Formation with Lenses

Consider an illuminated object at an axial distance o from a thin lens. A light ray from the top of the object passing through the center of the lens will not be deflected forming a straight line through the lens. A light ray from the top of the object and parallel to the axis will be deflected and pass through the focal point on the opposite side of the lens. An image is created where the two beams intersect. The distance i from the lens to the image plane, the focal length f, and the object length o are related by a simple expression,

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

(you can derive this result!). Verify this result experimentally as follows:

- Place a light-emitting diode at the left end of the optical bench. It operates on 5V, 12mA dc (red=plus, black=minus). On a vertical stand next to it place the metal strip with a transparent millimeter slide in its iris. The illuminated slide will be our object. To form an image use the lens held by a vertical twisted wire whose focal length is known from an earlier measurement. In order to form a real image the object distance has to satisfy $o > f$.

- Place a movable screen (blue cardboard) on the image side of the lens and observe that at a certain distance i a sharp image of the mm grating will be formed. It is advisable to calculate roughly where you expect the image to be formed so that you know where to place the screen. Measure i, o and f to verify the above expression. Repeat for several values of o so as to show that the expression holds in general.

  The size of the image I and object O are simply related by

$$\frac{I}{O} = \frac{i}{o} = \frac{f}{o-f}.$$

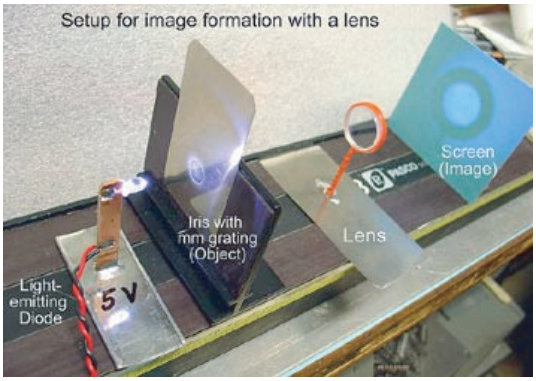

Figure 32: The setup for the image formation experiment. An image of an object is formed by focusing the light through an optical lens. For the object, you will use a 1mm grating, and the image will be the projection of the grating on a moveable screen. In the figure, the screen (image) is relatively close to the lens, however this may not necessarily be the case.

Magnification occurs if $i > o$ or $f < o < 2f$. For our setup, O is the spacing of the grating = 1mm and I is the spacing of the image of the grating formed on the screen. For $o = 2f$ one has $I = O$ but note from the dot on the slide that the image is inverted compared to the object. For $o > 2f$ the image is reduced. Verify this property by measuring at different values of o the image size with a ruler, knowing that the object size is a grating of 1mm/div. It may be easier to measure the width of several grid squares and divide to get an average measurement.

V.3 Analysis and Report Guidelines

In the previous experiments, you used statistical analysis to determine the best guess and uncertainty of repeatedly measured quantities, and also the fit parameters of multiple pairs of two measured quantities that exhibited some functional relationship. In the present laboratory assignment, you measured angles and distances with the aid of protractors and rulers. In this situation, our experimental uncertainty cannot be determined statistically, and is due the accuracy of the measurement device. For example, if you measure and object with a ruler that has 1mm spacings, you may visually see that the object lies closest the 20mm tick mark. Since the ruler only has accuracy up to 1mm, this means that the measured distance is most likely somewhere between 19.5mm and 20.5mm. Thus, when reporting this measured distance in your lab reports, it is reported as 20mm $\pm$0.5mm. This 0.5mm uncertainty can then be used in error propagation to determine the uncertainty in quantities which are calculated from this measured distance (see Taylor section 1.5 for further details on estimating uncertainties). **Note:** When doing error propagation make sure angles are written in radians! Degrees will not work in this case.

V.3.1 Lab report guidelines

Your lab report should contain the following sections

1. **Header**
    - Descriptive title

- Name, Date, and class information
- Name of your lab station partners

2. **Introduction**

   The introduction section explains, in your own words, the purpose of your experiment and how you will demonstrate this purpose. Try and be as brief as possible, yet still get your point across.

3. **Experimental Results**

   This section should briefly explain what was measured and how the data was collected (diagrams of the experimental set up may be helpful), present the raw data in graphical form (if possible) including uncertainty values with all numbers. Your graph should include labels on both axes that include units. Label your graphs as figure 1, figure 2, etc, so that you can refer to them in your text. In the text, explain the meaning of the variables on both axes and include an explanation that makes it clear to the reader how to interpret the information displayed on in the plots.

   your report should contain:

   - Describe the experimental set up/measurements from the Snell's law and total internal reflection section, and report the incident, reflected and transmitted angles you measured.
   - measured focal lengths of thick lenses, and discussion of spherical abberation and magnification.
   - Describe the experimental setup and report the measured focal lengths from thin lenses section
   - report the measured distances (i and o) and spacing size (I) of image/object formation section.

4. **Analysis**

   this section should present any calculations performed on the raw data, including uncertainties using propagation of errors. Example: if you are testing Ohm's Law $V = IR$, and you measured V and R, then here you calculate I. If you have a theoretical value for your calculated quantity (I in this example), state whether it is within your uncertainty range. Also, answer any questions proposed in the lab manual.

   you should have:

   - Using the measured angles from Snell's law, calculate the index of refraction n of the trapezoidal prism. Use the error propagation equation to determine the uncertainty in your calculated value for n.
   - Then, using this calculated index of refraction, determine a theoretical value for the angle of total internal reflection. Use the error propagation equation to determine your uncertainty in the theoretical Θ_c. Is your measured angle for TIR within your uncertainty of this theoretical value?
   - Calculate the magnification factor for your thick lens combination.
   - Using the measured focal lengths of thin lenses, verify the lens combination equation:

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{e}{f_1 f_2}.$$

- Verify that the position of an image formed by an object illuminated on a thin film is given by:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i},$$

and that the image size (grating spacing) is magnified or demagnified and whether it the image is inverted based on these relative positions.

5. **Conclusion**

State how your results demonstrate (or fail to demonstrate) the objectives you presented in the introduction section.

VI Laboratory 6: Diffraction and Interference

VI.1 Background information

In the present assignment, you will be performing experiments that demonstrate the wave-like nature of light. You will be taking advantage of some of the amazing characteristics of a laser, such as the coherence and the small beam divergence, in order to investigate double and multiple slit diffraction and interference. You will have one week to perform the experiment, and a report explaining your results will be due the following week.

VI.1.1 Single slit interference

In this laboratory exercise, you will investigate the interference patterns caused by a single slit, double slit, and multiple slit systems. First consider the single slit shown in Fig. 33. A coherent light source, such as a laser, shines onto a small slit with width b. We wish to describe the intensity of the light that is projected onto various positions of a distant screen. The various positions on the screen are described by the angle θ or the distance x as shown in the figure.

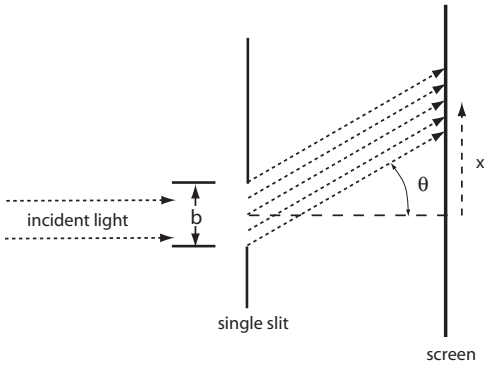

Figure 33: Single slit geometry: coherent light of wavelength λ is incident on the single slit from the left. The light passes through the slit of width b onto a screen. Constructive and destructive interference causes the intensity of the light to vary at different positions on the screen described by angle θ or position x.

The intensity at a specific point is determined by considering the light passing through the slit as made up of distinct rays of coherent light. Since the slit has a finite width, each ray will travel a different distance to reach this point on the screen. Since the light is coherent, there will be either constructive or destructive interference depending on the difference in path length, with respect to the wavelength of the light. This analysis yields the intensity is given by

$$I(\theta) = I_0 sinc(b \sin\theta/\lambda),$$

with $sinc(x) = \frac{\sin(\pi x)}{\pi x}$. This is plotted in figure 34. The graph shows that the intensity will have a central maximum peak at $\theta = 0$ or $x = 0$ and subsequent maxima and minima. The position of minima are given by:

$$b \sin\theta = m\lambda \tag{37}$$

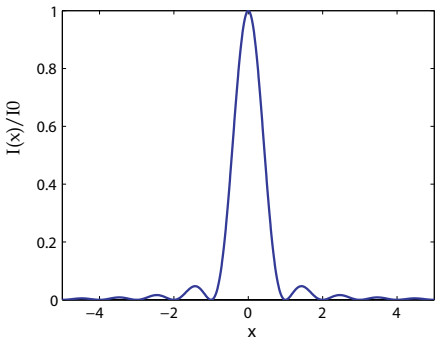

Figure 34: Normalized intensity of a single slit diffraction pattern is plotted as a function of position on the screen.

where $m = 1, 2, 3...$ We can convert this into distance x by using the approximation $\sin\theta \approx \tan\theta = x/D$, where D is the distance between the slit and the screen. Equation 38 becomes

$$b\frac{x}{D} = m\lambda \tag{38}$$

Thus, by measuring the distance between minima, you can determine the width single-slit.

VI.1.2 Double slit interference

For the double-slit case, we first analyze for the idealized situation. Figure 35 shows a coherent light source illuminating a double slit diffraction grating. The slits have a width b and seperation d. We assume that a single light ray passes through the slit and illuminates a position on a distant screen. As before, we wish to know the intensity of the light at various positions on the screen. This is given by the expression

$$I(\theta) = I_0 \cos^2\left[\frac{2\pi}{\lambda} d\cos\theta\right].$$

Since the light from the two slits will have to travel different distances, they will either constructively or destructively interfere. The angular positions of the maxima are given by:

$$d\sin\theta = m\lambda \tag{39}$$

The angular positions of the minima are given by

$$d\sin = (m + 1/2)\lambda \tag{40}$$

where $m = 0, 1, 2, 3...$ As in the single slit case, we can approximate $\sin\theta \approx \tan\theta$ to get the spacings of maxima and minima in terms of x.

In this analysis, we have neglected the fact that the slits themselves have a finite width, and thus will exhibit single slit diffraction. Therefore, in a real experiment like the one you are performing, the intensity on the screen will be a combination of both single slit and double slit diffraction. If the slit width b is explicitly accounted for, then the intensity $I(\theta)$ is a convolution of the results obtained individually for the single- and double-slit cases. That is,

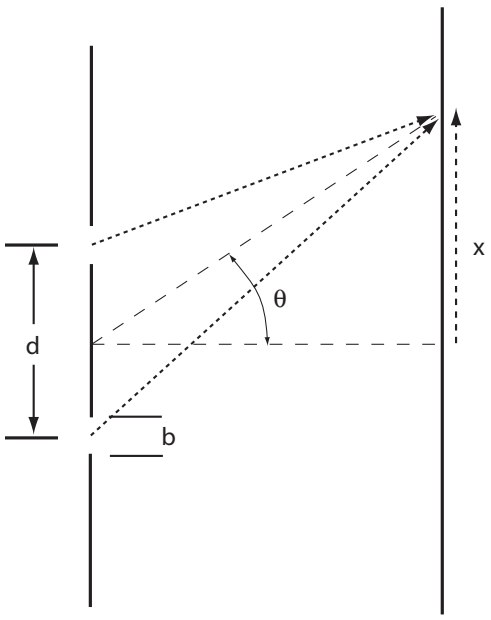

Figure 35: Coherent light source illuminates a screen after passing through a double slit diffraction grating. The slits have a width b and are separated by a distance d. We wish to know the intensity of at a position x on the screen.

$$I(\theta) \sim \cos^2\left[\frac{2\pi}{\lambda} d\cos\theta\right]\left[\frac{sin(\pi b \sin\theta/\ell)}{\pi b\theta/\ell}\right]^2, \tag{41}$$

with d the separation between slits, and b the width of each. The normalized intensity are shown in Fig. 36. The narrow maxima peaks are due to the double slit interference, and the overall envelope pattern is due to single slit diffraction. Thus, by measuring a diffraction pattern of a double slit grating, you can determine the slit width and slit separation.

VI.2 Experimental Setup

VI.2.1 Zeroing the Photometer

At your lab station you will find an optical bench. At one end of the bench there will be a laser while at the other end there will be a linear translator. Fitted into a small hole in the translator is a fiber-optic probe, which is connected to a photometer. Be very careful with the probe. Do not bend the probe in less than a 5cm radius at any given point (do not coil tighter than a 10 cm circle). Also, do not bend the probe within 8cm of either end.

The probe is attached to the photometer by slipping the optic output connector (BNC plug) of the probe over the input jack on the photometer. A quarter twist clockwise locks the probe to the photometer; a counterclockwise turn disengages it.

The following instructions are to zero the photometer. The lights in the classroom need to be turned off before you begin. First, turn the photometer off, and turn the SENSITIVITY knob all the way to the left, to the 1000 range setting. Next, carefully disconnect the Fiber Optic Probe from the photometer at the LIGHT PROBE INPUT jack. Completely cover up the end of the jack so that no light can enter (*e.g.*, use your thumb to cover the opening in the jack.) Turn the

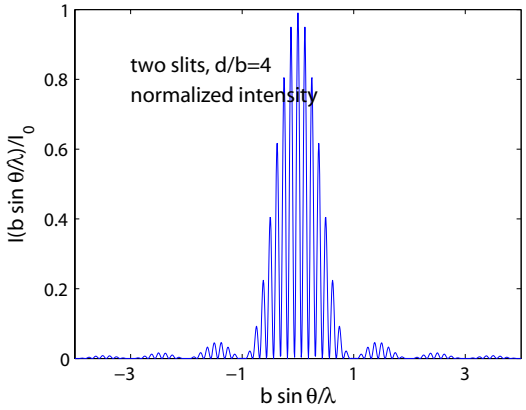

Figure 36: Intensity plot considering both single slit and double slit diffraction. The narrow maxima peaks and

photometer on. Turn the VARIABLE control fully clockwise and set the SENSITIVITY switch to the .1 range. Turn the ZERO ADJUST control until the METER reads zero. The instrument is now zeroed on all ranges (Do not remove your thumb yet). Set the SENSITIVITY to the 1000 range. Uncover the LIGHT PROBE INPUT and carefully reconnect the probe to the photometer.

The probe attenuates the light intensity reaching the selenium cell to approximately 6.5% of its value when the probe is not used. This makes measurements of absolute intensity impossible. However, for these experiments only the relative intensities are needed. When introducing light sources onto the probe, always start with the SENSITIVITY knob on a low sensitivity setting (the 1000 position is the least sensitivity). Shining an intense light source onto the probe when it is on a scale that is too sensitive can damage the photometer. After the new light source is aligned onto the detector, increase the sensitivity until the meter goes just barely off scale. Then reduce the VARIABLE setting until the needle is positioned near the maximum value on the scale.

VI.2.2 Measuring with the myDAQ

A diode laser illuminates a slit, which generates a diffraction pattern. Laser, slit and light detector are mounted on an optical bench and held in place by permanent magnets. The light intensity is measured with a light meter. The light meter is connected to an optical fiber that is mounted on a movable support. The carriage is moved transverse to the laser beam by rotating a threaded rod. The rod is connected to a linear potentiometer, which produces a voltage proportional to the position. Apply +5 V from a dc power supply via the red/black leads to the potentiometer. Obtain the variable voltage via the white leads. This voltage $(0...5V)$ is fed into CH0 of the myDAQ. The conversion of voltage vs. position can be obtained from a millimeter scale mounted inside the carriage housing.

The light intensity is obtained from the analog output in the rear of the photometer. The output voltage is negative, has a dc offset of about 0.15V and, unfortunately, a large 60Hz ripple is superimposed on the dc signal. When digitized the ac ripple creates a very noisy pattern. Therefore a low-pass RC filter ($RC = 0.5s$) has been inserted to remove the ac component. The output of this filter should be applied to CH1 of the myDAQ. Connect the high voltage side of the capacitor to +CH1 and the low voltage side (connected to the resistor) to -CH1. Since the typical voltage range is $x = 0.15...0.25V$ choose lower and upper limits of $\pm 0.5V$.

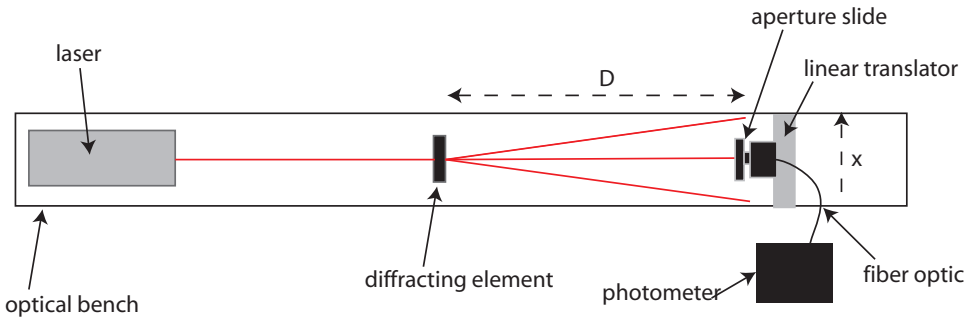

Figure 37: Schematic picture of the experimental setup and components.

In summary, the RED/BLACK leads of the potentiometer should be connected to 5V on the power supply; the white leads of the potentiometer should be connected to CH0 of the myDAQ; and the voltage across the capacitor on the back of the photometer should be connected to CH1 of the myDAQ.

VI.3 Procedure and Measurements

VI.3.1 Detector response

As a first exercise, you will measure the profile of the laser without any diffracting elements. After this practice run is completed, it should be easy to repeat the measurement with various diffraction gratings in place.

- **Aligning the laser:** Turn the sensitivity of the photodetector to its lowest value (range 1000). Position the fiber optics detector in the mid-range x=2.5cm. Next, adjust the vertical and horizontal position of the laser beam so as to shine directly into the fiber optic. Increase the sensitivity knob so that the needle reads somewhere between 4-8. Next, use the vertical and horizontal fine adjustment knobs on the laser to perfectly align the laser onto the fiber optic. As you are turning these knobs, observe the intensity on the photometer, and find the alignment that maximizes the intensity. The laser is now optimally aligned with the detector and it should not be moved for the remainder of the experiment. If the laser or the detector are accidentally moved, you will need to repeat this alignment procedure.

- **Insert the aperture** The optical fiber is too large to detect the small diffraction patterns you will produce. By placing a narrow slit just in front of the fiber we improve the spatial resolution of the detector. Place the slide labeled "photometer aperture" with single slits in front of the fiber detector so that the 0.1mm slit is positioned vertically in front of the fiber optic. Adjust the horizontal position of the slide to maximize the intensity signal (of course, the peak intensity will be reduced and you should increase the SENSITIVITY knob for nearly full scale response). Note: you are not studying the diffraction effects due to this aperture; it is used merely to reduce the amount of light hitting fiber optic. The aperture is now optimally aligned and should not be removed for the remainder of today's measurements.

- **scan of the laser beam profile**. The software should be set to "CH1 vs. CH0". Click the switch so that it is in "continuous" mode. Choose the limits on channel 0 to be $\pm 5V$ and on channel 1 $\pm 0.5V$ (these may need to be increased if your voltages exceed these limits), set

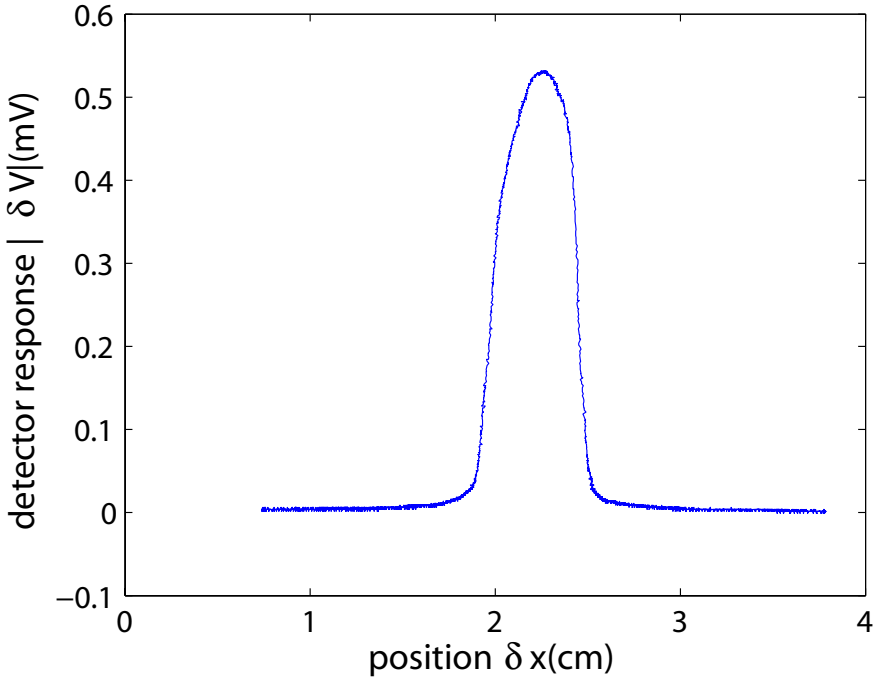

Figure 38: Graph of beam profile for a scan from x=1cm to x=4cm.

the sweep rate to 0.01 sec. Start at $x = 1cm$ and turn the handle counterclockwise at a slow rate (3sec/turn) so that the voltage change can fully respond to the low-pass filter response ($0.5s$). Start the measurement with the computer and slowly move the linear translator from x=1cm to x=4cm. Plot the beam profile in Excel by plotting the CH0 voltage on the x-axis and the CH1 voltage on the y-axis. Figure 38 shows what the graph should look like. Also, you can use this data set as your position to voltage conversion since your scan went from x=1cm to x=4cm. This portion will not be required in your report, however you may want save the data to file for voltage position conversion.

VI.3.2 Double-slit diffraction/interference patterns

With the 0.1mm slit in front of the fiber we have sufficient spatial resolution to measure the fine lines of a diffraction pattern of double slits. In the following exercises, you will determine the slit width and slit separation of various double slit diffraction gratings by measuring the diffraction pattern. To verify the procedure, you will first measure the diffraction pattern of a grating with known spacings. Then, you will measure the diffraction pattern with unknown parameters and determine them from your data. When measuring the diffraction pattern of these unknown gratings, consider the following question: as the double slit spacing increases, do the diffraction peaks become closer together or farther apart?

- Place a slide support between the laser and the detector. Measure the distance D to the detector (typically 35...45cm).Attach the slide with 4 different slits whose values are marked. Choose the double slit of width b=0.04mm and spacing d=0.125mm (this slit is labeled A). Position it so that the beam illuminates the slit.

- Look at the transmitted light through the slit by holding a sheet of paper at different distances from the slit. You will see the diffraction pattern, a sequence of lines in horizontal direction (for a vertical slit). Before taking data with the computer, scan the detector across the pattern and adjust the sensitivity to produce nearly full-scale response for the largest lines.
- Next, scan the whole profile sweeping $x = 1 \rightarrow 4$cm while recording with the myDAQ and save the data to file.

As discussed in the introduction section, adjacent maxima and minima are used to calculate the slit separation d and the overall envelop minima are used to determine the slit width b. From the measured positions of adjacent maxima, find the angular spacing, calculate the slit spacing d and compare with the value given. The laser wavelength is λ=670nm. Save the data to file to include in your lab report.

- In order to see the dependence of the diffraction pattern on slit spacing we use a slide containing three double slits of different line spacings. The values are not given to you and should be determined from your measurements. With the automated setup it is now quite fast and easy to repeat the measurement for these three double slits with unknown parameters. From your data evaluate the three slit spacings d. For the case of the largest slit spacing you can also determine the slit width b from the null in the envelope of the diffraction pattern. How does diffraction pattern change as the double slits become farther apart?

VI.3.3 Diffraction grating

You are supplied with a diffraction grating with 600 lines/mm. The identical lines are vertical and uniformly spaced. The grating, mounted to a 35mm slide frame, is illuminated with a red diode laser of wavelength λ=670nm and the diffraction pattern is observed in the transmitted light. The diffraction grating produces a diffraction spectrum that is easily seen by eye. Thus, for this portion of the assignment, you will be measuring the angles of adjacent maxima with a protractor and will not need the photometer setup. Your goal is to determine the angles of the maxima, and compare to the expected result.

- The best results are obtained by expanding the circular laser beam into a sheet beam. This is accomplished with a short glass tube (called a beam expander) placed horizontally into the laser beam, which produces a vertical sheet beam. The beam expander is shown in figure 39. To improve the resolution insert two vertical metal sheets between laser and beam expander. These form an adjustable vertical iris, which narrows the horizontal beam width.
- Pass the sheet beam through the diffraction grating. On the transmission side of the grating place on the sheet of paper with printed protractor horizontally with origin where the beams emerge. You should see several diffracted beams. Read off the angles and verify $d \sin\theta = n\lambda$. If the $\pm$ angles are unequal the diffraction pattern is probably not horizontal and you need to incline the paper plane.

VI.3.4 Dispersion of white light by a grating.

Use the collimated vertical sheet of light from the incandescent light bulb in the ray box (used in the geometric optics experiments) to illuminate the grating. Observe that the diffracted light

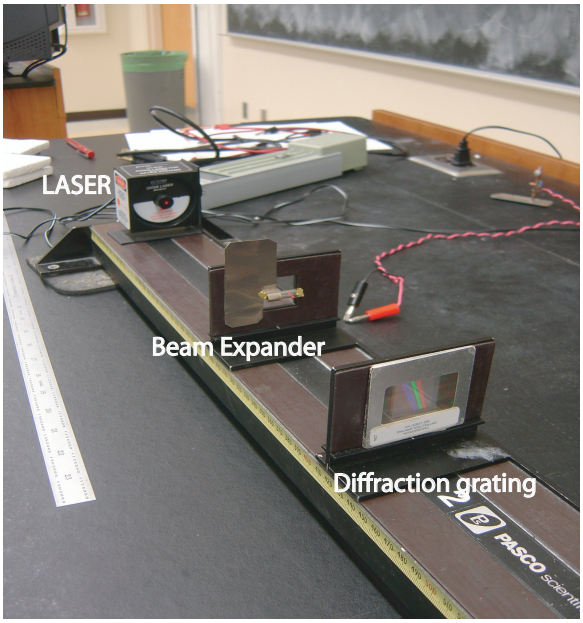

Figure 39: Laser, beam expander and diffraction grating. The beam expander is used to create a sheet beam of light.

beam separates into different colors. This is the principle of a spectrometer, an instrument to measure the spectrum of emission lines. Since $\sin\theta \sim \lambda$, the long wavelength colors (red) show a larger deflection than the short wavelength colors (blue). For the principle colors (red, green, and blue) measure the diffraction angles and calculate the wavelengths. Make sure you specify what the angles are in relation too.

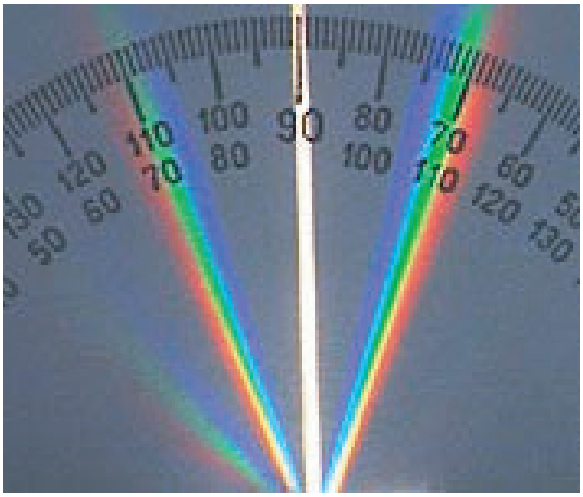

Figure 40: Separation by wavelength of white light by a diffraction grating.

VI.3.5 More experiments in using diffraction patterns to determine the size of scatterers

Next, you will perform three additional diffraction experiments. The first will be to determine the thickness of a hair by observing its diffraction pattern. The second will be to determine the bit spacing of a CD or DVD by observing the diffraction pattern. The third is to observe the diffraction pattern of a two dimensional array, and determine the thickness and separation of the wires forming the array. Record your results for all 3 in your notebook, however you will be required to comment on only one of these three in your report.

Human hair. You can measure the thickness of your hair from the diffraction pattern of a laser beam. Pull from your head a preferably straight, dark hair and place it vertically into the magnetic slide support, using two metal strips to hold it down. Shine the laser beam against the hair and observe the transmitted light at a distance of about L=90cm from the hair. Observe single-slit diffraction pattern on a white piece of paper and mark where the maxima peaks are located. The spacing δx between adjacent maxima or minima yields the thickness of the hair.

Two dimensional array Two line gratings crossed at right angles form both a horizontal and a vertical diffraction pattern. This can be seen by shining a laser beam (not expanded) through a fine mesh grid. It has been mounted in a slide between two papers with a hole in the center. Do not touch the fragile grid. Display the transmitted light on a white sheet of paper placed vertically at a distance of 50-100cm from the grid. Your diffraction pattern should look similar to Figure 41. Explain its features: The horizontal dot spacing yields the vertical line spacing and vice versa. Is

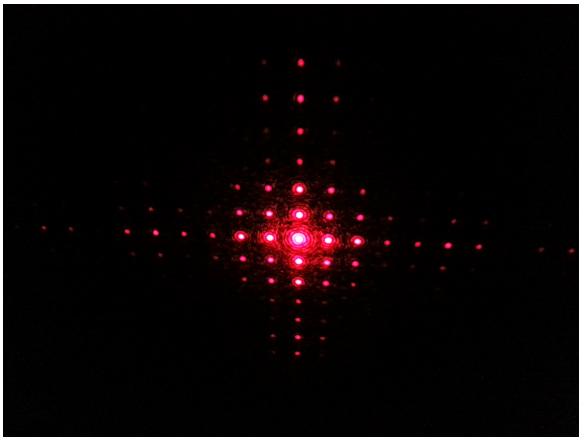

Figure 41: Pattern from the 2D diffraction grid.

the grid square? How many lines/inch does it have? From the envelope nulls you can determine the width between two wires (single-slit pattern).

VI.4 Analysis and Report Guidelines

VI.4.1 Double slit diffraction

As discussed in the introduction section, adjacent maxima or minima of the interference pattern of a double slit are used to calculate the slit separation d and the overall envelop minima are used to determine the slit width b. From the measured positions of adjacent maxima of the double slit with known parameters, find the angular spacing using equation 3, calculate the slit spacing d and compare with the value given. The laser wavelength is $\lambda = 670nm$.

From your data evaluate the three slit spacings d for the double slits with unknown parameters. For the case of the largest slit spacing you can also determine the slit width from the null in the envelope of the diffraction pattern. How does the diffraction pattern (distance between adjacent maxima) change with slit spacing d? When determining the slit spacing, repeat for as many adjacent maxima as you can to provide multiple measurements of the slit spacing. From these multiple values, find the average slit spacing and the uncertainty given by the standard deviation of the mean.

VI.4.2 Diffraction grating

A diffraction grating is many many slits equally spaced. How does this large number of slits affect the diffraction pattern? Use your observations by eye of the double slit and diffraction grating interference patterns to answer this question.

The distance between these sharp maxima is determined by equation 3. First, you measured the interference pattern of the diffraction grating with a laser as the light source. Since the laser has a wavelength of 670nm, use this measurement to verify the spacing of the diffraction grating slits: 600 lines/mm. Next, a white light source was used to illuminate the diffraction grating. White light is a combination of light of different wavelengths. Since the position of the diffraction peaks are dependent on wavelength (equation 3) the different colors will be diffracted a different amount. Use your measured spacing of the diffraction grating (previous part) and the diffraction angles of red and blue light to calculate the bandwidth (i.e. the range of frequencies) of visible light. For

the diffraction grating measurement, your uncertainty will be determined by your ability to read the protractor angles.

VI.4.3 Additional diffraction measurements

In the final portion of the exercise, you performed three experiments to measure the size of various diffracting objects. The three objects used are (1) a human hair (2) a CD or DVD and (3) a 2-dimensional grating. Pick one of these three measurements and, using your knowledge of diffraction and interference patterns, determine the size of the diffracting object.

VI.4.4 Lab report guidelines

Your lab report should contain the following sections

1. **Header**
   - Descriptive title
   - Name, Date, and class information
   - Name of your lab station partners
2. **Introduction**

   The introduction section explains, in your own words, the purpose of your experiment and how you will demonstrate this purpose. Try and be as brief as possible, yet still get your point across.
3. **Experimental Results**

   This section should briefly explain what was measured and how the data was collected (diagrams of the experimental set up may be helpful), present the raw data in graphical form (if possible) including uncertainty values with all numbers. Your graph should include labels on both axes that include units. Label your graphs as figure 1, figure 2, etc, so that you can refer to them in your text. In the text, explain the meaning of the variables on both axes and include an explanation that makes it clear to the reader how to interpret the information displayed on in the plots.

   your report should contain:
   - Explain how you measured diffraction/interference patterns, and include a graph of diffraction pattern for the double slit with *known parameters*.
   - Graphs of diffraction patterns for double slits with *unknown parameters*.
   - For the diffraction grating: describe the the diffraction patterns as observed by eye, and report the measured angles for both laser light and white light.
   - described the observed diffraction pattern as observed by eye and report the measured angles for *1 of the three* '"additional diffraction"' measurements.
4. **Analysis**

   this section should present any calculations performed on the raw data, including uncertainties using propagation of errors. Example: if you are testing Ohm's Law $V = IR$, and you measured V and R, then here you calculate I. If you have a theoretical value for your calculated

quantity (I in this example), state whether it is within your uncertainty range. Also, answer any questions proposed in the lab manual.

you should have:

- Verify the slit width and slit spacing of the double slit with known parameters. Is the provided value for the slit spacing within your experimental uncertainty?
- Determine the slit spacing (of all three) and slit width (of only the largest) of the double slits with unknown parameters. Does increasing the slit spacing decrease or increase the distance between adjacent maxima?
- For the diffraction grating: verify the slit spacing from your measurements with the laser, and calculate the bandwidth (spread in frequencies) of visible light.
- Determine the size of one of the three additional diffraction measurements.

5. **Conclusion**

   State how your results demonstrate (or fail to demonstrate) the objectives you presented in the introduction section.